A RUDIMENTARY TREATISE

ON

LAND AND ENGINEERING SURVEYING

For Students and Practical Use

By T. BAKER, C.E.

AUTHOR OF "SUBTERRANEOUS SURVEYING," "STATICS AND DYNAMICS," "MECHANISM AND CONSTRUCTION OF MACHINES," "MENSURATION," "MATHEMATICAL THEORY OF THE STEAM ENGINE," ETC. ETC.

Fifteenth Edition, Revised and Corrected

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Capio Lumen

LONDON

CROSBY LOCKWOOD AND SON

7, STATIONERS' HALL COURT, LUDGATE HILL

1891

PREFACE.

Land Surveying is a branch of the Mathematics applied to practical purposes, and had its origin, it is said, in Egypt, more than 1000 years before the Christian era, where the annual inundations of the Nile, and the consequent large deposits of mud, destroyed the land marks of the different proprietors. It therefore became necessary to determine these land marks by measurement, or to lay out the proper quantities of land claimed by the several proprietors, irrespective of their land marks, thus destroyed. Hence the origin of the science of *geometry*, so called from its being compounded of two Greek words signifying "*to measure the earth.*"

But, notwithstanding the early origin of this science, in as far as it is applied to land surveying, it has received comparatively very little improvement almost up to the present time. The writers of extensive works on this subject being chiefly practical men, unacquainted with modern analysis, and some of them even ignorant of geometry, have successively produced works which are little more than mere copies of those previously published, their examples being in all cases simple and profuse without variety or elegance, the invariable result of such labours being a large volume immeasurably behind the requirements of the subject as well as the science of modern times.

In the following work, the author trusts, it will be found that he has avoided such profusion and want of variety in the subjects proposed, and made, at the same time, the most important parts sufficiently clear to those students who have not had an opportunity of studying geometry and the higher branches of mathematics,—the demonstrations of all rules and formulæ not previously published being given in notes, apart from the practical matter; while the demonstrations of all rules previously given in

various works of geometry and analytical trigonometry, are omitted, to avoid increasing the size of the work, which is divided into Two Parts, viz., LAND SURVEYING and ENGINEERING SURVEYING, and each part into six chapters; on each of which it will be proper to make a few observations.

CHAPTERS I. and II., On Practical Geometry and Surveying by the Chain and Cross, are of too simple a character to admit of improvement, the author's object in these being condensation and clearness, with a sufficiency of examples to introduce the subject.

CHAPTER III. On Surveying with the Chain only, contains several new Problems, among which may be named Problem IV., which furnishes an original and concise method of finding the width of a large river. Problem VII. gives three methods of surveying fields of from five to seven sides, with only five chain lines, with examples of the numerous lines adopted in old methods; and Problem IX. gives a method of surveying a small estate of six fields by either five or four chain lines, with the method of proving the positions of straight fences; which positions all previous authors have determined by the crossing of two chain lines, or by prolonging each straight fence to two chain lines; which methods constitute no check on wrong entries in the Field Book. This Chapter concludes with several specimens of laying out main lines of extensive surveys, which occurred in the author's practice.

In CHAPTER IV. are given engravings and descriptions of the most efficient drawing and surveying instruments, which are chiefly taken from HEATHER'S TREATISE ON MATHEMATICAL INSTRUMENTS, in the Rudimentary Series. In this Chapter Rodham's method of keeping the Field Book, invented about fifty years ago, is given, with slight alteration, on a folding plate, and the plan of an estate, to which the field book refers.

CHAPTER V. contains several surveys, chiefly by the Theodolite, including surveys for railways and other engineering purposes. In this Chapter directions for town-surveys are rather prominently introduced.

CHAPTER VI., on dividing and inclosing land, commons,

&c., has been almost entirely remodelled, several formulæ, never before published, being given for the expeditious laying out and dividing land of uniform or variable value, the demonstrations of which are given in the notes, or among the Formulæ in Part II., Chapter VI. This Chapter concludes Part I.; which, it will be seen, treats exclusively on land surveying.

PART II. of this work may with propriety be called modern, if we except Chapter I. on Levelling, which has been practised above a century, firstly for canals, and secondly for drainage, roads, and railways; however, no good treatise on this subject appeared till that by Mr. Simms, whose plates and examples, being in the publishers' possession, were adopted by the author in this work, as well as some parts of Mr. Simms' accompanying explanations of the subject. In this Chapter the author has also availed himself of HEATHER'S TREATISE ON MATHEMATICAL INSTRUMENTS for the plates of the levels, &c., and some parts of their descriptions.

CHAPTER II. treats of the various methods required for laying out railway curves in the ground; these methods were invented by the author about thirty years ago, and he trusts that it will be found that he has treated this subject in an improved and practical manner: the investigations of the additional formulæ here required are given in the notes.

In CHAPTER III. the methods of setting out the widths of railway cuttings, on all varieties of sloping and undulating ground, are carried out chiefly by original formulæ, Mr. Simms also wrote on this subject, in his work on LEVELLING, but without giving any formulæ; these methods may therefore be considered as entirely newly modelled.

CHAPTER IV. is on tunnelling, on the setting out of which very little has been written by any author. The author has, he trusts, given clear and practical methods for this purpose, whether the tunnel be straight or curved, with copious notes, chiefly extracted from DEMPSEY'S PRACTICAL RAILWAY ENGINEER.

CHAPTER V. is on the author's concise and original method of finding the contents of the earthwork of rail-

ways, almost entirely without calculation, which was invented by him many years ago.

CHAPTER VI. contains a collection of problems and formulæ of utility in land and engineering surveying, the investigation of which are either given or referred to in other works, with a collection of unsolved problems original and select, for the exercise of students. At the end of this Chapter is given PAMBOUR'S FORMULÆ for the super-elevation of the exterior rail in railway curves.

The work concludes with an Appendix, in which are given the dimensions of the famous tubular bridges, and several of the principal railway viaducts of various constructions, which the late general extension of railways has called into existence in this kingdom, and various useful addenda.

From the preceding analysis of the contents of this work, as well as from an inspection of its several details, and a comparison of these with the works of other authors, it will at once be seen that of the twelve Chapters of which this work is composed, four may be safely claimed by the author as having been originally drawn up by him, viz., Chapters II., III., IV. and V[illegible] Part II., while in all the other Chapters, excepting I. and II. of Part I., considerable additions and improvements have also been made.

NOTE TO THE FIFTEENTH EDITION.

Mr. Baker's Rudimentary Treatise on Land and Engineering Surveying, like his Treatise on Mensuration, has become a text-book in many of the principal schools in this country and in the colonies.

The work in its present form has had the advantage of the careful revision of the late Professor Young, formerly of Belfast College, under whose direction several errors which had crept in were removed, and other improvements made.

In issuing the present edition, opportunity has been taken to substitute the last three paragraphs now appearing on p. 221 for the paragraph with which, in previous editions, that page was concluded, and which was found to be erroneous in some particulars.

February, 1891.

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LAND AND ENGINEERING SURVEYING.

PART I.

LAND SURVEYING.

CHAPTER I.

PREVIOUS to commencing the various subjects of Land and Engineering Surveying, it will be necessary to give a clear view of Practical Geometry, which is especially requisite for those who are unacquainted with this branch, as well as those parts of the Mathematics which are equivalent to it.

PRACTICAL GEOMETRY.

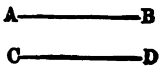
DEFINITIONS.

1. *A point* has no dimensions, neither length, breadth, nor thickness.

2. *A line* has length only, as A.

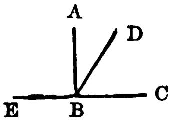
3. *A surface or plane* has length and breadth, as B.

4. *A right or straight line* lies wholly in the same direction, as A B.



5. *Parallel lines* are always at the same distance from each other, and never meet when prolonged, as A B and C D.

6. *An angle* is formed by the meeting of two lines, as A C, C B. It is called the angle A C B, the letter at the angular point C being read in the middle.

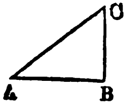


7. *A right angle* is formed by one right line standing erect or perpendicular to another; thus, A B C is a right angle, as is also A B E.

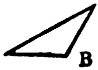
8. *An acute angle* is less than a right angle, as D B C.

9. *An obtuse angle* is greater than a right angle, as D B E.

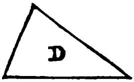
10. *A plane triangle* is a space included by three right lines, and has three angles.



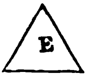
11. *A right angled triangle* has one right angle, as A B C. The side A C, opposite the right angle, is called the hypothenuse; the sides A B and B C are respectively called the base and perpendicular.



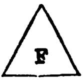
12. *An obtuse angled triangle* has one obtuse angle, as the angle at B.



13. *An acute angled triangle* has all its three angles acute, as D.

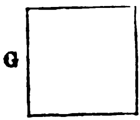


14. *An equilateral triangle* has three equal sides, and three equal angles, as E.



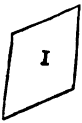
15. *An isosceles triangle* has two equal sides, and the third side greater or less than each of the equal sides as F.

16. *A quadrilateral figure* is a space bounded by four right lines, and has four angles; when its opposite sides are equal, it is called a *parallelogram.*

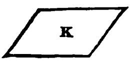


17. *A square* has all its sides equal, and all its angles right angles, as G.

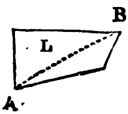
18. *A rectangle* is a right angled parallelogram, whose length exceeds its breadth, as B, (see figure to definition 2).



19. *A rhombus* is a parallelogram having all its sides and each pair of its opposite angles equal, as I.

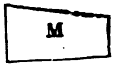


20. *A rhomboid* is a parallelogram having its opposite sides and angles equal, as K.



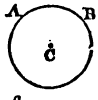
21. *A trapezium* is bounded by four straight lines, no two of which are parallel to each other, as L. A line connecting any two of its angles is called the *diagonal,* as A B.

22. *A trapezoid* is a quadrilateral, having two of its opposite sides parallel, and the remaining two not, as M.



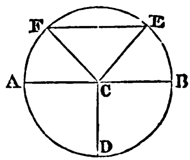
23. *Polygons* have more than four sides, and receive particular names, according to the number of their sides. Thus, a *pentagon* has five sides; a *hexagon*, six; a *heptagon*, seven; an *octagon*, eight; &c. They are called regular polygons, when all their sides and angles are equal, otherwise irregular polygons.

24. *A circle* is a plain figure, bounded by a curve line, called the circumference, which is everywhere equidistant from a point C within, called the centre.



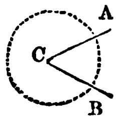
25. *An arc of a circle* is a part of the circumference, as A B.

26. The *diameter of a circle* is a straight line A B, passing through the centre C, and dividing the circle into two equal parts, each of which is called a *semicircle.* Half the diameter A C or CB is called the *radius.* If a radius CD be drawn at right angles to A B, it will divide the semicircle into two equal parts, each of which is called a *quadrant,* or one fourth of a circle. A *chord* is a right line joining the extremities of an arc, as F E. It divides the circle into two unequal parts called *segments.* If the radii C F, C E be drawn, the space, bounded by these radii and the arc F E, will be the *sector of a circle.*



27. The circumference of every circle is supposed to be divided into 360 equal parts, called degrees, and each degree into 60 minutes, each minute into 60 seconds, &c. Hence a semicircle contains 180 degrees, and a quadrant 90 degrees.

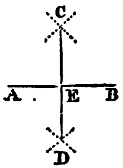
28. *The measure of an angle* is an arc of any circle, contained between the two lines which form the angle, the angular point being the centre; and it is estimated by the number of degrees contained in that arc:—thus the arc A B, the centre of which is C, is the measure of the angle A C B. If the angle A C B contain 42 degrees, 29 minutes, and 48 seconds it is thus written $42° 29' 48''$.



PROBLEMS IN PRACTICAL GEOMETRY.

(In solving the five following problems only a pair of common compasses and a straight edge are required; the problems beyond the fifth require a scale of equal parts; and the two last a line of chords: all of which will be found in a common case of instruments.)

PROBLEM I.

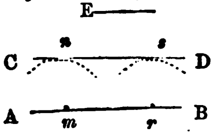


To divide a given straight line A B *into two equal parts.*

From the centres A and B, with any radius, or opening of the compasses, greater than half A B, describe two arcs, cutting each other in C and D; draw C D, and it will cut A B in the middle point E.

PROBLEM II.

At a given distance E, *to draw a straight line* C D, *parallel to a given straight line* A B.

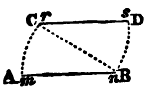


From any two points *m* and *r*, in the line A B, with a distance equal to E, describe the arcs *n* and *s*:—draw C D to touch these arcs, without cutting them, and it will be the parallel required.

NOTE. This problem, as well as the following one, is usually performed by an instrument called *the parallel ruler.*

PROBLEM III.

Through a given point r, to draw a straight line CD *parallel to a given straight line* A B.

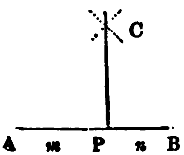


From any point *n* in the line A B, with the distance *n r*, describe the arc *r m*:—from centre *r*, with the same radius, describe the arc *n s*:—take the arc *m r* in the compasses, and apply it from *n* to *s*:—through *r* and *s* draw C D, which is the parallel required.

PROBLEM IV.

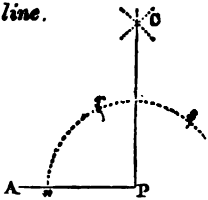
From a given point P *in a straight line* A B *to erect a perpendicular.*

1 *When the point is in or near the middle of the line.*



On each side of the point P take any two equal distances, P *m*, P *n*; from the points *m* and *n*, as centres, with any radius greater than P *m*, describe two arcs cutting each other in C; through C, draw C P, and it will be the perpendicular required.

2. *When the point* P *is at the end of the line.*



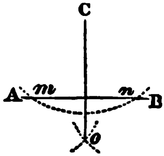
With the centre P, and any radius, describe the arc $n\ r\ s$;—then with the same radius, and taking n and r as centres, cut off the equal arcs $n\ r$ and $r\ s$:—again, with centres r and s, describe arcs intersecting in O:—draw O P, and it will be the perpendicular required.

NOTE. This problem and the following one are usually done with an instrument called *the square.*

PROBLEM V.

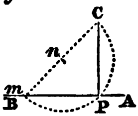
From a given point C *to let fall a perpendicular to a given line.*

1 *When the point is nearly opposite the middle of the line.*



From C, as a centre, describe an arc to cut A B in m and n;—with centres m and n, and the same or any other radius, describe arcs intersecting in o: through C and o draw C o, the perpendicular required.

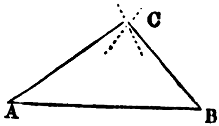
2. *When the point is nearly opposite the end of the line.*



From C draw any line C m to meet B A, in any point m;—bisect C m in n, and with the centre n, and radius Cn, or $m\,n$, describe an arc cutting B A in P. Draw C P for the perpendicular required.

PROBLEM VI.

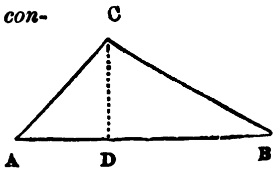
To construct a triangle with three given right lines, any two of which must be greater than the third. (Euc. I. 22.)



Let the three given lines be 5, 4 and 3 yards. From any scale of equal parts lay off the base A B = 5 yards; with the centre A and radius A C = 4 yards, describe an arc; with centre B and radius C B = 3 yards, describe another arc cutting the former arc in C:—draw A C and C B; then A B C is the triangle required.

PROBLEM VII.

Given the base and perpendicular, with the place of the latter on the base, to construct the triangle.

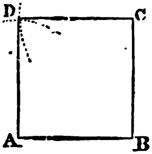


Let the base A B = 7, the perpendicular C D = 3, and the distance A D = 2 chains. Make A B = 7 and AD = 2;—at D erect the

perpendicular D C, which make = 3:—draw A C and C B; then A B C is the triangle required.

PROBLEM VIII.

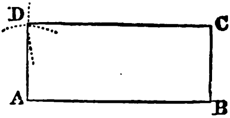
To describe a square, whose side shall be of a given length.



Let the given line A B be three chains. At the end B of the given line erect the perpendicular B C, (by Prob. IV. 2.) which make = AB:—with A and C as centres, and radius A B, describe arcs cutting each other in D: draw A D, D C and the square will be completed.

PROBLEM IX.

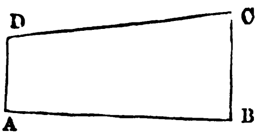
To describe a rectangled parallelogram having a given length and breadth.



Let the length A B = 5 chains, and the breadth B C = 2. At B erect the perpendicular BC, and make it = 2:—with the centre A and radius B C describe an arc; and with centre C and radius A B, describe another arc, cutting the former in D:—join A D, D C to complete the rectangle.

PROBLEM X.

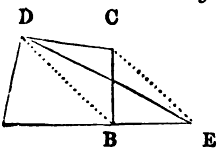
The base and two perpendiculars being given to construct a trapezoid



Let the base A B = 6, and the perpendiculars A D and B C, 2 and 3 chains respectively. Draw the perpendiculars A D, D C, as given above, and join D C, thus completing the trapezoid.

PROBLEM XI.

To make a triangle equal to a given trapezium A B C D.

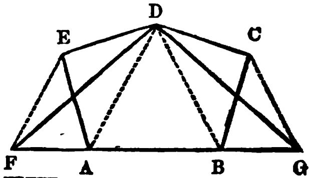


Draw the diagonal D B, and C E parallel to it, meeting A B prolonged in E:—join D E; then shall the triangle A D E be equal to the trapezium A B C D.

PROBLEM XII.

To make a triangle equal to the figure A B C D E A.

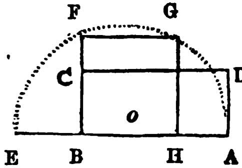
Draw the diagonals D A, D B, and the lines E F, C G,



parallel to them, meeting the base A B, produced both ways, in F and G:—join D F, D G; then the triangle D F G will be equal to the given figure A B C D E A.

Problem XIII.

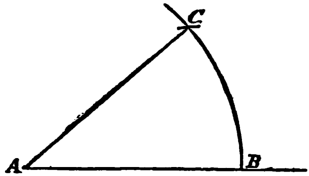
To make a square equal to a given rectangle A B C D.



Produce one side A B till B E be equal to B C:—bisect A E in *o*; on which as a centre, with radius A *o*, describe a semicircle, and prolong B C to meet it in F:—on B F describe the square B F G H, and it will be equal to the rectangle A B C D, as required.

Problem XIV.

To set off an angle to contain a given number of degrees.



Let the angle be required to contain 41 degrees. Open the compasses to the extent of 60° upon the line of chords, and, setting one foot upon A, with this extent, describe an arc cutting A B in B; then taking the extent of 41° from the same line of chords, set it off from B to C; join A C; then B A C is the angle required.

Problem XV.

To measure an angle contained by two straight lines.
(*See last figure.*)

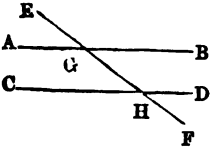
Let A B, A C contain the angle to be measured. Open the compasses to the extent of 60°, as before, on the line of chords, and with this radius describe the arc B C, cutting A B, A C produced, if necessary, in B and C; then extend the compasses from B to C, and this extent, applied to the line of chords, will reach to 41°, the required measure of the angle B A C.

A right angle, or perpendicular, may be laid off by extending the arc B C, and setting off the extent of 90° thereon. Also an angle greater than 90° may be laid off, by still further extending the arc, and laying the excess of the arc above 90°, from the end of the 90th degree.

Note. Angles are more correctly and expeditiously laid off and measured by an instrument called the protractor, to be hereafter described.

GEOMETRICAL THEOREMS.

(*Necessary to be known by Surveyors.*)



Theorem I.

Angles vertically opposite are equal:—thus the angle A G E = angle H G B, and E G B = A G H. (Euc. I. 15.)

Theorem II.

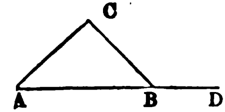
(*See last figure.*)

A right line E F, *cutting two parallel right lines* A B, C D, *makes the alternate angle equal, &c.* :—thus the angles A G H, G H D are equal; also the exterior angle E G B is equal to the interior and opposite G H D. (Euc. I. 29.)

Theorem III.

The greatest side of every triangle is opposite the greatest angle. (Euc. I. 18.)

Theorem IV.



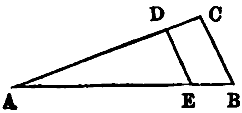
Let the side A B *of the triangle* A B C *be produced to* D, *the exterior angle* C B D *is equal to the interior angles at* A *and* C; *also the three interior angles of the triangle are equal to two right angles.* (Euc. I. 32.) Whence any two angles of a triangle being given the third becomes known.

Theorem V.

(*See figure to Definition* 11.)

Let A B C be a right angled triangle, having a right angle at B; then, *the square on the side* A C *is equal to the sum of the square on the sides* A B, B C. (Euc. I. 47.) Whence any two sides of a right angled triangle being given the third becomes known.

Theorem VI.



In any triangle A B C, let D E be drawn parallel to one of its sides, CB; then, A B *is to* A E *as* B C *is to* D E; and the triangles are said to be similar. (Euc. VI. 2.)

Theorem VII.

(*See last figure.*)

Let A B C, A E D be similar triangles; then, *the triangle* A B C *is to the triangle* A E D *as the square of* A B *is to the square of* A E: that is, similar triangles are to one another in the duplicate ratio of their homologous sides. (Euc. VI. 19.)

THEOREM VIII.

All similar figures are to one another as the squares of their homologous, or like, sides. (Euc. VI. 20.)

THEOREM IX.

All similar solids are to one another as the cubes of their like linear dimensions.

CHAPTER II.

DESCRIPTION OF INSTRUMENTS USED FOR MEASURING AND PLANNING SMALL SURVEYS.

THE CHAIN.

THE chain, usually called Gunter's chain, is almost generally used in the British dominions, for measuring the distances required in a survey. It is 66 feet, or 4 poles, in length, and is divided into 100 links, which are joined by rings. The length of each link, together with half the rings connecting it with the adjoining links, is consequently $\frac{66}{100}$ of a foot, or $\frac{66 \times 12}{100} = 7{\cdot}92$ inches. At every tenth link from each end is attached a piece of brass with notches; that at the tenth link has one notch, that at the 20th two notches, that at the 30th three, that at the 40th four, the middle of the chain, or the 50th link being marked with a large round piece of brass; hence, any distance on the chain may be readily counted. Part of the first link, at each end, is formed into a large ring for the purpose of holding it with the hand.

The chain acquires extension by much use, it should, therefore, be frequently examined, and adjusted to the proper length by taking out some of the rings between the links: for this purpose, chains having three rings between each link are to be preferred to those having only two.

THE OFFSET STAFF.

The offset staff is used to measure short distances, called offsets; hence its name. It is usually ten links in length, the links being numbered thereon with the figs. 1, 2, 3, &c. It is usually pointed with iron at one end, for the purpose of fixing it in the ground, as an object for ranging lines, for marking stations, &c.

THE CROSS.

THE cross is an instrument used by surveyors to erect perpendiculars. It is usually a round piece of sycamore, box, or

mahogany, about four inches in diameter, with two folding sights at right angles to each other, or more commonly with two fine grooves sawed at right angles to each other, which answer the purpose of sights. It is sometimes fixed on a staff, of convenient length for use, pointed with iron at the bottom, that it may be fixed firmly in the ground: but it is found more commodious in practice to have a small pocket cross, which may be readily fitted to the offset-staff, either by an iron spike on the cross being inserted in a hole made in the offset-staff, or the offset-staff being passed through a hole made in the cross, to about the eighth link from the piked end, at which place the staff must be shouldered, that the cross may rest firmly.

DIRECTIONS FOR MEASURING LINES ON THE GROUND.

Besides the instruments already described, ten arrows must be provided, about 12 inches long, pointed at the end, so as to be readily pressed into the ground, and turned at the other end, so as to form a ring to serve for a handle.

In using the chain, marks are to be set up at the extremities of the line to be measured, as well as at its intermediate points, if its extremities cannot be seen from one another, on account of hills, woods, hedges, or other obstructions. Two persons are then required by the surveyor to perform the measurement. The chain leader starts with the ten arrows in his left hand, and one end of the chain in his right; while the follower remains at the starting point, who, looking at the staff or staves, that mark the line to be measured, directs the leader to extend the chain in the direction of the staff or staves. The leader then puts down one of his arrows, and proceeds a second chain's length in the same direction, while the follower comes up to the arrow first put down. A second arrow being now put down by the leader, the first is taken up by the follower; and the same operation is repeated till the leader has expended all his arrows. Ten chains, or 1000 links, having now been measured and noted in the field book, the follower returns the ten arrows to the leader, and the same operation is repeated as often as necessary. When the leader arrives at the end of the line, the number of arrows in the follower's hand shew the number of chains measured since the last exchange of arrows noted in the field book, and the number of links extending from the last arrow to the mark or staff at the extremity of the line, being also added, gives the entire measurement of the line. Thus, if the arrows have been exchanged seven times, and

7000
600
83
7683

if the follower have six arrows, and from the arrow last put down to the end of the line be 83 links, the whole measurement will be 7683 links, or 76 chains 83 links, which is usually written thus—76·83 chains, the two last figures being decimals of a chain.

In using the chain, care must be taken to stretch it always with the same tension, as it will extend by much use, and will therefore require to be examined occasionally, and shortened, if necessary. But a good chain may be used several days, on tolerably smooth ground, without any material extension.

The surveyor must mark, or cause to be marked, every station on the line, while it is being measured, with a staff or cross on the ground, entering its distance in the field book.

When a survey is made for a finished plan, all remarkable objects should be noted down; as buildings, roads, rivers, ponds, footpaths, gates, &c.

The boundary of the estate measured ought to be carefully observed. If the ditch be outside the boundary fence, it usually belongs to the estate, and *vice versa;* although this is not universally the case; therefore, inquiry ought to be made with respect to the real boundary.

In some places five links from the hedge-posts or roots of the quickwood are allowed for the breadth of the ditch, but this breadth varies to as far as even ten links, especially in swampy countries.

All ditches and fences must be measured with the fields to which they belong, when the full quantity on the plan is required: but when the growing crops only are to be measured, only so much as is occupied by the crops.

Instruments for Laying Down or Plotting Surveys.

THE COMMON DRAWING COMPASSES.

This instrument consists of two legs moveable about a joint, so that the points at the extremities of the legs may be set at any required distance from one another: it is used to transfer and measure distances, and to describe arcs and circles.

THE HAIR COMPASSES.

The hair compasses ought to be used where greater accuracy is required in transferring distances, than can be obtained by the set of the joint of the common compasses. In the hair compasses the upper part of one of the steel points is formed into a bent spring, which, being fastened at one extremity to

the leg of the compasses almost close up to the joint, is held at the other end by a screw. A groove is formed in the shank, which receives the spring when screwed up tight; and by turning the screw backwards the steel point may be gradually allowed to be pulled backwarks by the spring, and may again be gradually pulled forwards by the screw being turned forwards.

Fig. 1

Fig. 2.

Fig. 3.

Fig. 1, represents these compasses when shut; fig. 2, represents them open, with the screw turned backwards, and the steel point *p*, in consequence, moved backwards by its spring *s*, from the position represented by the dotted lines, which it would have when screwed tight up. Fig. 3, represents a key, of which the two points fit into the two holes seen in the nut *n* of the joint; and by turning this nut the joint is made stiffer or easier at pleasure.

To take a distance with the hair compasses.—Open them as nearly as you can to the required distance, set the fixed leg on the point from which the distance is to be taken, and make the extremity of the other leg coincide accurately with the other end of the required distance, by turning the screw.

NOTE. There are several other kinds of compasses, used for planning; as those with moveable points, for the introduction of black lead pencils or ink points, beam compasses for taking large distances, proportional compasses, &c., the uses of which are easily learned. (See *Heather's Treatise on Mathematical Instruments.*)

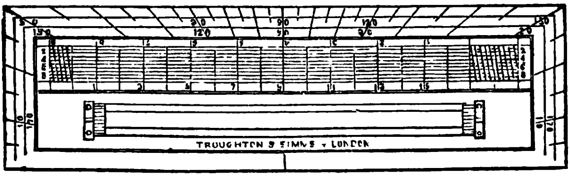
PLOTTING SCALES.

Plotting scales, also called feather-edged scales, are straight rulers, usually about 12 inches long. Each ruler has scales of equal parts, decimally divided, placed on its edges, which are made sloping, so that the extremities of the strokes marking the divisions lie close to the paper. The primary divisions represent chains, and the subdivisions ten links each, the intermediate links being determined by the eye. Plotting scales

may be procured in sets, each with a different number of chains to the inch. They are usually made of ivory or box, and each provided with a small scale called an offset scale for laying down the offsets. In using these scales, the first division or zero, on the plotting scale, is placed coincident with the beginning of the line to be plotted, and so as just to touch that line with the feather edge: the end of the offset scale is then placed in contact with the edge of the plotting scale; and thus the offsets may be expeditiously pricked off, for which purpose an instrument called a pricker is used, but a hard black lead pencil with a fine point, is greatly to be preferred, as it does not injure the paper.

THE PROTRACTOR.

The instrument represented in the annexed figure, is usually supplied with a pocket case of instruments. It is made of ivory, 6 inches long and 1¾ broad. On the face of the instrument, round three of its edges, which are feathered, the protractor is formed for readily setting off angles.



In using the protractor, the fourth edge, which is quite plain, with the exception of a single stroke in the middle, is to be made to coincide with the line, from which the angle is to be set off, and the stroke in the middle to coincide with the angular point in the line, at which the angle is to be set off; a mark is then to be made with a fine pointed black lead pencil, or with a pricking point, at the point on the paper which coincides with the stroke on the protractor marked with the number of degrees in the angle required to be drawn; and the protractor being now removed, a straight line is to be drawn from the angular point in the given line to the point thus marked off. The instrument has, on the same face, two diagonal scales, (which are now little used by surveyors,) and on the opposite face, scales of equal parts, &c.

The vernier scale and circular protractor, the uses of which will be hereafter described, are best adapted to laying down extensive surveys, where great accuracy is required.

PLANNING SURVEYS.

In planning or plotting surveys, the upper part of the paper or book, on which the plan is made, should always, if possible, be the north. The chain lines, buildings, fences, &c., ought first to be drawn with a fine black lead pencil: the first should then be dotted with ink, and the latter neatly drawn. Great care is required in the construction of the plan, when the dimensions are to be measured therefrom with the scale. The scale should never be more than three chains to an inch, for when the parts of a plan are large, the dimensions may be taken with greater accuracy. After having found the content of the field or fields, &c., of which any plan consists, it may be laid down by any scale to give it a more convenient size.

THE FIELD BOOK.

The method now generally adopted in setting down field notes, and which has long been found to be the best in practice, is to begin at the bottom of the page and write upwards.

Each page of the book is usually divided into three columns. The middle column is for distances measured on the chain line, at which hedges are crossed, or offsets, stations, or other marks are made; and in the right and left columns, those offsets, marks, and any other necessary observations thereon, must be entered, according as they are situated on the right or left of the chain line.

The crossing of roads, rivers, hedges, &c., are, by some surveyors, shewn in the field book, by lines drawn across the middle column at the distances where they are crossed, and by others these crossings are shewn by lines drawn on part of the right and left hand columns, opposite the distances where they are crossed by the chain line; and buildings, turns of fences, corners of fields, to which offsets are taken, are usually shewn by lines sketched in a similar situation to the middle column, as the fences, buildings, &c., have to the chain line. Thus a representation of the chief objects in the survey may be sketched in the field, which will give essential assistance in laying down the plan. The stations are usually numbered, for the sake of reference, and marked thus ⊙. The bearing of the first main line is usually taken by surveyors, from which the position of the plan with respect to the north is determined. This may be done by a common pocket compass, where great accuracy is not required: but this will be more fully discussed in treating of surveys by the theodolite, further on.

R. of ⊙ 2, and L. of ⊙ 5, &c., denote that the following lines are measured to the right of station 2, and to the left of station 5, respectively.

To Survey with the Chain and Cross.

An acre of land is equal to 10 square chains, that is 10 chains in length and one in breadth, or 1000 links in length and 100 in breadth; an acre, therefore, contains 100,000 square links, as per table of square measure below. Hence the contents in square links are, in the following examples, divided by 100,000, or what is the same thing 5 figures to the right are cut off for decimals, the figures remaining on the left being acres. The decimals are then multiplied by 4 for roods, and again by 40 for poles.

The following tables exhibit the number of chains and links in the different units of lineal measure, and the number of square chains and links in the different units of square measure.

A TABLE OF LINEAR MEASURES.

Links.	Feet.	Yards.	Poles.	Chains.	Furlongs.	Mile.
25	16½	5½	1			
100	66	22	4	1		
1,000	660	220	40	10	1	
8,000	5,280	1,760	320	80	8	1

A TABLE OF SQUARE MEASURES.

Sq. Links	Sq. Feet.	Sq. Yards.	Sq. Poles or Perches	Sq.Chs.	Roods.	Acres.	Sq. Mile.
625	272¼	30¼	1				
10,000	4,356	484	16	1			
25,000	10,890	1,210	40	2½	1		
100,000	43,560	4,840	160	10	4	1	
64,000,000	27,878,400	3,097,600	102,400	6,400	2,560	640	1

Problem I.

SQUARE AND RECTANGULAR FIELDS.

Square and rectangular fields seldom occur in the practice of the surveyor; small plots of ground of these forms, however, frequently present themselves.

Fix the cross in a corner of the square or rectangular piece

of ground, and if the sides be at right angles, measure one of them, and enter its length in the field book. This must be repeated at every side and angle; and if all the angles are found to be right and all the sides equal, the piece of ground is square; but, if only the opposite sides are equal, it is rectangular.

TO FIND THE CONTENT.

Rule. Multiply the length by the breadth in links, and the product will be the contents in square links, which reduce to acres, rods, and poles by the preceding table for square measure, as in the following examples.

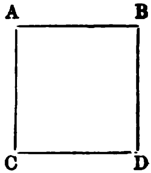
If the dimensions be given in yards, divide the product by 4840 for acres, &c.

EXAMPLES.

1. What is the area of the square A B C D, whose side is 478 links?

Here the length and breadth are equal, hence

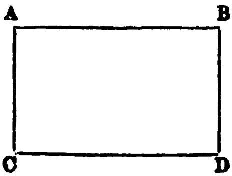
478
478
———
3824
3346
1912
———
2·28484
4
———
1·13936
40
———
5·57440



Area $2a.\ 1r.\ 5\frac{1}{2}p.$

2. The length of the rectangular field ABCD, is 2450, and its breadth 1226 links; required the plan and area.

2450
1226
———
14700
490
490
245
———
30·03700
4
———
0·14800
40
———
5·92000



Area $30a.\ 0r.\ 6p.$ nearly.

3. Required the area of a square, in acres, the side of which is 132 yards.

$$\frac{132 \times 132}{4840} = 3{\cdot}6 \text{ acres} = 3a.\ 2r.\ 16p. \text{ the area.}$$

4. What is the area of a rectangle whose length is 2470 links, and its breadth 1114 links?

$$2470 \times 1114 = 27{\cdot}51580 = 27a.\ 2r.\ 2\tfrac{1}{2}p. \text{ the area.}$$

5. The length of a rectangular field is 324 yards, and its breadth 235 what is its content in acres?

Ans. 15*a*. 2*r*. 37*p*

6. What is the area of a rectangular pleasure ground, the length of which is 960 and its breadth 125 links?

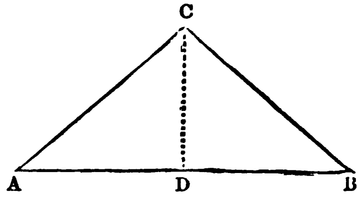
Ans. 1*a*. 0*r*. 32*p*.

Problem II.

TRIANGULAR FIELDS.

1. Let A B C be a triangle, of which the survey, plan, and content are required.

Set up poles or marks at the angles A, B, and C, and measure from A towards B, and when at or near D, try with the cross for the place of the perpendicular C D; plant the cross, and turn it till the marks A and B can be seen through one of the grooves; then look through the other groove, and, if the mark at C can be seen through it, the cross is in the right place for the perpendicular; if not, move the cross backward or forward till the three marks can be seen as before directed. Suppose the distance A D to be 625 links, and the whole A B, to be 1257 links; return to D, and measure the perpendicular D C, which suppose to be 628 links, thus completing the survey of the triangle.



CONSTRUCTION.

From a scale of equal parts, or plotting scale, lay off the base A B, = 1257 links; on which take A D, = 625 links; at D erect the perpendicular D C, which make = 628 links; join A C, C B, then A B C is the plan of the triangle.

TO FIND THE CONTENT.

RULE. Multiply the base by the perpendicular, and half the product will be the area.

EXAMPLES.

1. The dimensions being the same as found above, required the content.

Ans. $1257 \times 628 \div 2 = 3{\cdot}94698$ acres $= 3a.\ 3r.\ 32p.$

2. The distance from the beginning of the base to the place of the perpendicular is 375 links, the whole base 954, and the perpendicular 246; what is the area of the triangle?

$954 \times 246 \div 2 = 1{\cdot}17342 = 1a.\ 0r.\ 27\frac{1}{2}p.$ the content.

3. Measuring the base of a triangle the place of the perpendicular was found at 863 links, and its length 645; the whole base was 1434 links; required the plan and area.

Area. $4a.\ 2r.\ 20p.$

PROBLEM III.

FIELDS IN THE FORM OF TRAPEZIUMS.

Fields in this form are usually divided into two triangles by a diagonal, which is a base to both the triangles.

Let A B C D be a field in the form of a trapezium, the plan and area of which is required.

Measure from A towards C; and let the place of the perpendicular mB be at 5·52, and its length 3·76, also let the place of the perpendicular nD be at 11·82, and its length 3·44, and the length of the whole diagonal A C be 13·91 chains, which completes the survey: but it is usual also to measure the other diagonal B D for a proof line, which is found to be 9·56 chains.

NOTE 1. The construction of each of the two triangles, forming the trapezium, is the same as the construction given to the first example in PROB. II.

NOTE 2. The longer of the two diagonals should always be selected for the base of the two triangles, forming the trapezium, for sometimes the perpendicular will not fall on the shorter diagonal, without its being prolonged; and when this is the case with both diagonals, one of the sides may be taken for a base, or two of the sides, if necessary.

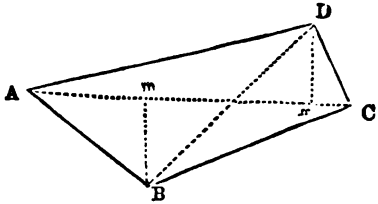
TO FIND THE CONTENT.

RULE. Multiply the sum of the two perpendiculars by the diagonal, and half the product will be the content.

EXAMPLES.

1. Let the measurement of a trapezium be as above found, required the content.

```
      344
      376
      ———
      720
     1391
    —————
    27820
    9737
   ——————
2)10·01520
   ——————
   5·00760
         4
   ——————
   0·03040
        40
   ——————
   1·21600
```



Ans. *5a. 0r. 1p.*

2. From the following notes, plan and find the content of a field.

Perpendiculars on left.	Base or Station Line.	Perpendiculars on right.
	to ⊙ C	
	3250	
	2504	1046 D
B 1278	1272	
Begin	at ⊙ A	and range West.

Content. 37*a.* 3*r.* 2*p.*

3. Give the plan and area of a field from the following notes.

	A C	
	872	
B 652	731	
	423	545 C
Begin at	⊙ A	and range E.

Area. 5*a.* 0*r.* 35*p.*

ANOTHER METHOD.

A four-sided field may frequently be surveyed by dividing it into two triangles and a trapezoid, by perpendiculars on its longest side.

TO FIND THE CONTENT.

Rule. Multiply the sum of the two perpendiculars by their distance on the base line for the double area. The double areas of the two triangles must be found as in Prob. II., and both be

added to the double area of the trapezoid; the sum being divided by 2, and five figures marked off for decimals, will give the content required.

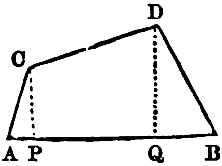
EXAMPLES

1. Required the survey and area of the following field.

Measure the base A B, and put down in the field book the distances of P and Q, where the perpendiculars rise, &c., as below.

	to ⊙ D.
	A B = 1097
Q D = 595	A Q = 745
P C = 352	A P = 110
Perpen.	From ⊙ A go E.

Triangle A C P.	Triangle Q D B.	Trapezoid P C D Q.
352	595	352 } perp.
110	352	595 }
38720	1190	947 sum.
	2975	635 = P Q.
	1785	4735
	209440	2841
		5682
		601345
		38720
		209440
		2)8·49505
		4·24752
		4
		·99008
		40
Area. $4a.\ 0r.\ 39\frac{3}{5}p.$		39·6032



2. Required the plans and areas of two fields from the following notes.

	A B	
	1169	
E	615	339 D.
G	234	461 C.
From	⊙ A go	W.

	A B	
	1448	
D 513	1102	E.
C 683	436	G.
From	⊙ A	go E.

PROBLEM IV.

TO SURVEY FIELDS CONTAINED BY MORE THAN FOUR SIDES.

Fields or plots of ground bounded by more than four sides, may be surveyed by dividing them into trapeziums and triangles.—Thus, a field of five sides may be divided into a trapezium and a triangle; of six sides, into two trapeziums; of seven sides into two trapeziums and a triangle; &c.

TO FIND THE CONTENT.

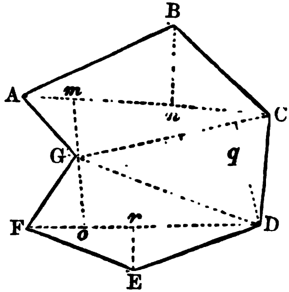
RULE. By the two last Problems, find the double areas of

each trapezium and triangle in the field; add all the double areas together, and half their sum will be the content.

EXAMPLES.

1. Lay down a field and find its area from the following dimensions.

	to ⊙ D	
	520	
r	288	80 E
G 120	206	*o*
	Go to ⊙ F.	
	to ⊙ G	
	440	
D 230	152	*q*
	L. of ⊙ C.	
	to ⊙ C	
	550	
B 180	410	*n*
m	135	130 G
Begin	at ⊙ A	range E.



CONSTRUCTION.

The above field is divided into trapeziums ABCG, GDEF and the triangle GCD.—Draw the diagonal AC, which make = 550 links; at 135 and 410 set off the perpendiculars *m*G = 130, and *n*B = 180 links respectively; join AB, BC, CG, and GA, and the first trapezium will be completed. Then on CG, lay off C*q* = 152, and draw the perpendicular *q*D = 230; join CD, DG, and the triangle is completed. Lastly, with centre G and radius *o*G = 120 describe an arc; and with centre D and radius *o*D = 314 (= 520 − 206) describe another arc, intersecting the former in *o*: through *o* draw the diagonal DF = 520 links, upon which, at 288 links, draw the perpendicular *r*E; join DE, EF, FG, and the figure will be completed.

130	440	120	Double areas.
180	230	80	170500 trap. ABCG.
			101200 tri. CDG.
310	13200	200	104000 trap. DEFG.
550	88	520	
			2)3·75700
15500	101200	104000	
1550			1·87850 = 1a. 3r. 20½p.
170500			

2. Required the plans and areas of two fields from the following dimensions.

First Field.

	to ⊙ A	
	504	Base.
E 98	233	
Return	to ⊙ B	
	to ⊙ D	
	673	Diag.
C 207	472	
	427	268 B.
Begin	at ⊙ A	range W.

Area 1*a.* 3*r.* 15*p.*

Second Field.

	to ⊙ F	
	970	Diag.
	520	181 B.
E 290	413	
	R. of ⊙ D	
	to ⊙ D	
	744	Diag.
C 161	386	
	303	333 B.
Begin	at ⊙ A	range W.

Area 4*a.* 0*r.* 19½*p.*

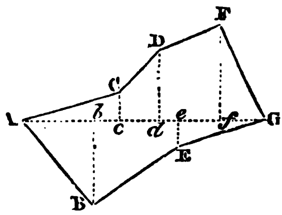
ANOTHER METHOD.

A small piece of land, having several sides, may sometimes be most conveniently measured by taking one diagonal, and upon it erecting perpendiculars to all the angles on each side of it. The piece will thus be divided into right angled triangles and trapezoids, the areas of which must be calculated as in the two last Problems.

EXAMPLES.

1. Required the plan and area of a field from the following notes.

	to ⊙ G	
	1020	
F 470	890	*f*
e	610	50 E.
D 320	585	*d*
C 70	440	*c*
b	315	350 B
Begin	at ⊙ A	go E.



NOTE. The method of planning the above field is sufficiently clear, from the preceding field-notes and from what has been already done.

Triangle A C *c*.	Trape. C *c d* D.	Trape. D *d f* F.	Tri. F *f* G.	Tri. A *b* B.
A *c* = 440	D *d* = 320	D *d* = 320	*f* G = 130	A *b* = 315
C *c* = 70	C *c* = 70	F *f* = 470	F *f* = 470	B *b* = 350
30800	sum = 390	sum = 790	9100	15750
	c d = 145	*d f* = 305	52	945
	13050	3950	61100	110250
	435	2370		
	56550	240950		

Trapezoid B *b e* E.	Triangle E *e* G.
B *b* = 350	G *e* = 410
E *e* = 50	E *e* = 50
sum = 400	20500
b e = 295	
118000	

Double areas.
30800
56550
240950
61100
110250
118000
20500

2)638150

3·19075

= 3*a*. 0*r*. 30½*p*. Area.

2. Lay down two pieces of ground, and find their areas, from the following dimensions.

First piece.

	to ⊙ R	
o	3401	*o*
I 579	2930	*i*
h	2736	730 H
g	2110	762 G
F 288	1972	*f*
e	1588	436 E
d	1030	740 D
C 500	888	*c*
b	300	550 B
o	000	*c*
Begin	at ⊙ A	go E.

Area. 31*a*. 0*r*. 7*p*.

Second piece.

	to ⊙ L	
M 920	4300	*o*
k	3340	590 K
I 790	3060	*i*
h	2690	320 H
G 1340	2550	*g*
F 800	1760	*f*
e	1560	540 E
d	930	300 D
C 910	610	*c*
o	000	600 B
Begin	at ⊙ A	go W.

Area. 54*a*. 2*r*. 11¾*p*.

Problem V.

FIELDS INCLUDED BY ANY NUMBER OF CROOKED OR CURVED SIDES.

When a field or estate is bounded by crooked fences, a line must be measured as near to each of them, as the angles or bends will admit; in doing which an offset must be taken to each corner or bend in the fence. When the fences are curved, these offsets must be taken so as neither to exclude nor include any of the land belonging to the ground to be measured. The offsets or perpendiculars thus erected, will divide the whole offset space into right angled triangles and trapezoids, the areas of which may be found as already shewn.

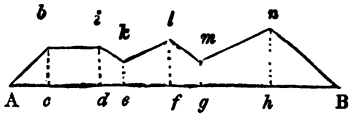
Note 1. When the offsets are short, that is, not greatly exceeding a chain in length, their places on the line may be found by laying the offset staff at right angles to the chain, as nearly as can be judged by the eye; but when the offsets are large, and correctness is required, their places must be found by the cross, and measured by the chain.

Note 2. The quickest method of laying down offsets, is, by laying the feather edge of the plotting scale against the base or chain line, and sliding the offset scale along the feather edge to the several distances of the offsets and pricking off their lengths, corresponding to their several distances.

Note 3. Unskilful surveyors usually add all the offsets taken on one line togethor and divide the sum by their number for a mean breadth; but this method is very erroneous, especially where the offsets vary greatly in length, and should therefore be avoided where great accuracy is required.

EXAMPLES.

1. Required the plan and content of a right-lined piece of ground by offsets, from the following notes.



	to ⊙ B	
o	955	
n 91	785	*h*
m 57	634	*g*
l 88	510	*f*
k 70	340	*e*
i 84	220	*d*
b 62	45	*c*
o	00	
Begin	at ⊙ A	range E.

Ac = 45	*cb* = 62	*di* = 84	*ek* = 70	*fl* = 88
cb = 62	*di* = 84	*ek* = 70	*fl* = 88	*gm* = 57
90	146	154	158	145
270	*cd* = 175	*de* = 120	*ef* = 170	*fg* = 124
2790	730	18480	11060	580
	1022		158	290
	146			145
			26860	
	25550			17980

		Double areas.
gm = 57	*h*B = 170	2790
hn = 91	*hn* = 91	25550
		18480
148	170	26860
gh' = 151	1530	17980
		22348
148	15470	15470
740		
148		2)1·29478
22348		0·64739 = 0*a.* 2*r.* 23½*p.*

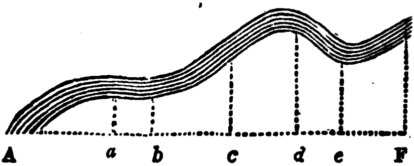
Calculation by the erroneous method (See Note 3).

00	955	= A B.
62	56½	= mean breadth.
84		
70	5730	
88	4775	
57	477	
91		
00	0·53957	= 0*a*. 2*r*. 6*p*. Content by this method, *which is 17½ perches too little.* For this method is always erroneous except when the offsets stand at equal distances from one another, and when the first and last offsets are both 0.
8)452		
56½		

Some omit all the offsets that are 0, dividing the sum of the offsets by the number of real offsets; by this method we shall have

6)452	955
	75⅓
75⅓	
	4775
	6685
	318
	0·71943 = 0*a.* 2*r.* 35*p.*, *which is 11½ poles too much.*

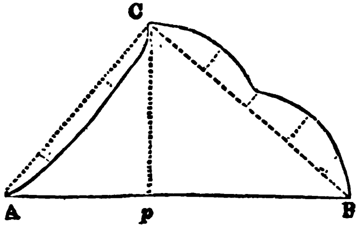
2. To lay down a crooked piece of land, adjoining a river from the following notes.



	to ⊙ F	
197	720	
163	600	
226	500	
139	350	
80	200	
74	100	
0	000	[E.
Begin	at ⊙ A	range

The content is found by the same method as in the preceding example.

3. Plan and find the area of a field from the subjoined notes.



	to ⊙ A	
0	480	
37	350	
28	160	
0	000	
	L. of ⊙ C	
	to ⊙ C	
	585	0
	450	57
	320	40
	200	72
	100	47
	000	0
	L. of ⊙ B	

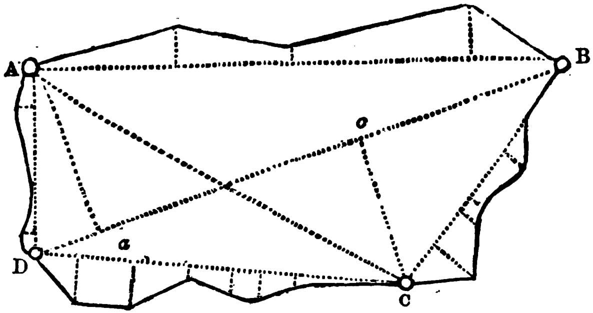
	to ⊙ B	
	743	
C 382	290	p
From	⊙ A	go E.

Having found the area of the triangle ABC, the areas of the offsets on the line B C must be added thereto, and the sum of the areas of the *insets* on the line CA must be subtracted from the sum, and the remainder will be the content of the field.

NOTE. The area of the triangle A B C may be found, when the measurement of all three sides are given, (which is the case in the present example,) either by calculation, as shall hereafter be shewn, or by measuring the perpendicular from the plan, which, as already shewn, may be laid down from the three sides of the main triangle.

4. The notes, plan, and content of the following field, the

lines for the measurement of which are sketched out as below, are required.



Here the double area of the trapezium A B C D is found, as in Problem III, to which the double areas of the several offsets on the lines A B, B C, C D, D A must be added, and the sum will be the double area of the whole.

NOTE. A proof line, as A C should be measured, that the accuracy of the work may be insured:—but often pieces of ground are measured with the chain and cross that are never laid down, in this case great care should be taken in entering the field-notes, and in making the calculation.

5. Required the plan and area of a field from the following notes.

	To fence.	
80	2024	
84	1900	to ⊙ D
108	1200	
66	800	
24	520	
60	300	
130	000	
From	⊙ C	go E.
	To fence.	
50	1230	
70	1100	to ⊙ C
60	800	
20	480	
44	300	
30	000	
From	⊙ B	go N.
	To fence.	
66	2180	
90	2090	to ⊙ B
112	1600	
80	1000	
96	600	
60	000	
Begin	at ⊙ A	go W

	to ⊙ C	
	2310	diag.
B 990	1810	
	720	1040 D.
From	⊙ A	go N. W
	To fence.	
96	1320	
106	1260	to ⊙ A.
80	1000	
50	760	
100	400	
124	000	
From	⊙ D	go S.

Double areas.

4689300 Trap. A B C D

392220	offsets on	A B
131920		B C
325016		C D
235280		D A

2)57·73736

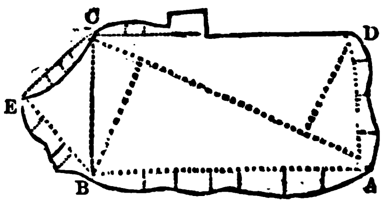
28·86868 = 28*a*. 3*r*. 19*p*.
The whole area.

NOTE. The field in this case has also four sides, like the one proposed to be measured in the last example. Fields of this kind are often required to be measured in the rural districts, to ascertain the quantity of a growing crop, as grain, hay, turnips, &c., when sold by the acre; also to find the quantity of reaping, mowing, planting, &c.; in these cases the plan is seldom or never required, and the measurement is only taken as far as the growing crop extends, leaving out the hedges, ditches, and all other waste or other ground, not occupied by the crop in question.

6. Give the plan and area of a field from the following notes.

	to ⊙ A	
0	1134	
64	916	
134	728	
48	470	
86	245	
0	000	
	R. of ⊙ D	
	to ⊙ D	
0	2236	
258 + 0	930	
160 + 60	620	
130	400	
84	200	
0	000	
	R. of ⊙ C	
	to ⊙ C	
	1168	diag.
E 586	656	
	R. of ⊙ B	
	to ⊙ B	
0	2346	
70	2000	
88	1800	
156	1500	
92	1200	
170	800	
84	400	
0	000	
From	⊙ A	go W

	to ⊙ B	
	884	0
	690	148
	512	50
	230	112
	000	0
	L. of ⊙ E	
	to ⊙ E	
0	774	
86	592	
118	400	
72	200	
0	000	
	L. of ⊙ C	
	to ⊙ C	
	2588	diag.
B 1060	2100	
	2000	
	1000	
	580	970 D
	R. of ⊙ A	



Double areas.

5253640	Trapezium A B C D	
684448	Triangle B E C	
443420	Offsets on	A B
249580		C D
149352		D A
135400		E B

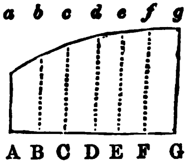
6915840
107220 Insets on C E to be subtracted.

2)68·08620

34·04310 = 34*a*. 0*r*. 7*p*. nearly. Area.

PROBLEM VI.

TO FIND THE AREA OF A SEGMENT OF A CIRCLE, OR ANY OTHER CURVILINEAL FIGURE BY EQUIDISTANT OFFSETS OR ORDINATES.



RULE. If a right line A G be divided into any number of equal parts, AB, BC, CD, &c., and at the points of division perpendiculars be erected, A *a*, B *b*, C *c*, &c., to the curve *a b c d e f g*; then to the sum of the first and last offsets, add four times the sum of all the even offsets, and twice the sum of all the odd offsets, not including the first and last; multiply the sum by the common distance of the offsets, and one-third of the product will be the area, recollecting that the second, fourth, &c., are the even offsets, and the third, fifth, &c., are the odd offsets.

NOTE. If any portion of the figure is not included by an even number of offsets, its area must be found separately and added to the area found by the Rule.

EXAMPLES.

1. Required the plan and area of a piece of land measured by equidistant offsets or ordinates, from the following dimensions. (See last figure.)

	to ⊙ G	
g 450	600	G
f 442	500	F
e 430	400	E
d 406	300	D
c 376	200	C
b 306	100	B
a 212	000	A
From	⊙ A	go E.

First and last offsets.
1st. = 212 = A*a*.
Last = 450 = G*g*.
———
662 sum.

Even offsets.
2nd. = 306 = B*b*
4th. = 406 = D*d*.
6th. = 442 = F*f*.
———
1154 sum.
4
——— [sum.
4616 quadruple

Odd offsets.
3rd. = 376 = C*c*.
5th. = 430 = E*e*.
———
806 sum.
2
———
1612 twice sum
4616
662
———
6890 sum total.
100 com. distance.
———
3)6·89000
———

Area. 2*a*. 1*r*. 7½*p*. = 2·29666

2. Required the plan and area of a piece of land, measured by equidistant offsets, the dimensions being as given below.

	to ⊙ B	
	1071	35 *c*
a	1000	48½ *d*
	900	69½
	800	87½
	700	103
	600	115
	500	124
	400	130
	300	132
	200	134
	100	131
	000	126½ *b*
From	⊙ A	go W.

175 First and last offsets.
2238 Four times even offsets.
933 Twice odd offsets.
———
3346 sum total.
100 common distance.
———
3)3·34600
———
1·11533 Area A *a d b*.
2864 Trap. *a* B *c d*.
———
1·14397 = 1*a*. 0*r*. 23*p*. Area.

Here the area of the trapezoid *a* B *c d* is found separately and added, agreeable to the preceding note.

3. Plan and find the area of a piece of ground from the equidistant offsets given below.

	to ⊙ B	
115	600	90
126	500	96
130	400	91
121	300	82
110	200	69
93	100	43
70	000	24
From	⊙ A	go E.

Here the piece of ground is curved on two sides, the base line A B passing nearly in the middle space between the two curves; in this case the sum of the offsets on each side of every distance must be considered as one offset, in finding the area.

4. Required the areas of two fields, the ends of which are straight and parallel, and the side curved by the following equidistant offsets.

	to ⊙ B	
216	1200	200
150	1100	240
116	1000	256
100	900	255
77	800	260
65	700	259
67	600	250
80	500	229
100	400	202
120	300	180
160	200	144
202	100	95
277	000	50
From	⊙ A	go N.

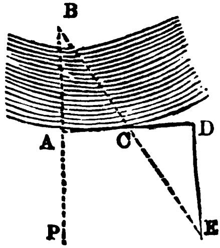
Area *3a. 3r. 35p.*

	to ⊙ B	
210	1097	209
180	1000	157
150	900	112
120	800	89
93	700	80
96	600	60
95	500	57
130	400	35
184	300	10
240	200	0
315	100	0
390	000	0
From	⊙ A	go W

Area *2a. 2r. 15p.*

PROBLEM VII.

TO MEASURE A LINE ACROSS A WIDE RIVER.



Let the annexed figure be a river, which is required to be crossed by the chain-line P B. Fix, or cause to be fixed, a pole or mark at B, at or near the margin of the river, in the line to be measured; erect the perpendicular A D, measuring A C and C D of any equal lengths; at D erect the perpendicular D E; on arriving at E, in the direction of B C produced, the distance D E will be equal to A B, the required breadth of the river.

From the arrangement of the lines in the figure, it is evident that the triangles C A B, C D E are equiangular, and since A C was made = C D, the triangles are equal in all respects, and consequently A B = D E.

NOTE. We have thus given in sufficient detail the mode of surveying with the cross, which, though not much used by experienced surveyors, is a simple instrument, and its use readily understood by students. This method is, therefore, a proper introduction to the higher branches of surveying; besides, in rural districts, villages, &c., few surveyors use the more expensive instrument, the chain and cross being found quite sufficient to measure the quantities of growing crops, and other such small surveys as may be there required.

CHAPTER III.

LAND SURVEYING BY THE CHAIN ONLY.

THIS method of surveying has long been adopted by experienced surveyors; who have found it, in general, more accurate and expeditious, as well as better adapted to laying down extensive surveys, especially where no serious obstructions from woodlands, water, buildings, &c., exist; the use of the cross, in this method, being entirely excluded by some surveyors, and by others only used for secondary purposes, as for taking occasionally long offsets, or for squaring of lines obstructed by buildings, water, &c. Instead of the cross some use the Optical Square for these purposes; which will be hereafter described; while some erect perpendiculars with the chain only, as shall be shewn in the following Preliminary Problems.

The fundamental lines of surveys of this kind usually form a large triangle, or several triangles, abutting from one common base, which ought, if possible, to extend throughout the whole length of the survey. The sides of the triangle, or triangles, must run as near as possible to the external and internal fences of the estate, or district, to be surveyed; the sides of each triangle being connected by one or more lines, running anywhere within the triangle, to determine the accuracy of the work. These lines are called proof or tie-lines; and where the estate to be surveyed contains a great number of inclosures, the proof-lines may almost always be found available in determining the positions of some of these inclosures. Where a great number of lines run within the main triangle, they are called secondary lines, and are usually numbered for the sake of reference. Some surveyors number the stations, or extremities of the lines; but the former method is here recommended. In small surveys, for preliminary instructions, the numbering of the lines is unnecessary, the stations being referred to by the letters of the alphabet, as already done in Chap. II.

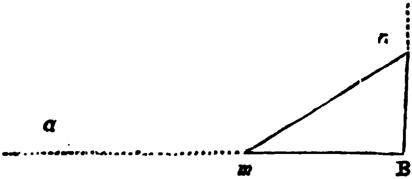
PRELIMINARY PROBLEMS.

PROBLEM I.

TO ERECT A PERPENDICULAR WITH THE CHAIN.

Let *a* B be a chain-line, and A B the extended chain. It is required to erect a perpendicular to *a* B at B. Fix the end of the chain to the ground with an arrow at B; fix also the 80th link of the chain, reckoning from B, at *m*, 40 links from B; 80 links of the chain now lying slack between B and *m*. Take

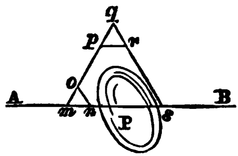
hold of the 30th link of the chain from B, and extend it till it take the position B*nm*, the portions B *n*, *m n* of the chain being pulled tight; then shall B *n* be perpendicular* to the chain-line *a* B, and may be extended to any length required.



This method of erecting a perpendicular, though not so expeditious as that by the cross or optical square, is quite sufficient for those surveyors, who scarcely once require a perpendicular in their operations for weeks together; thus avoiding the inconvenience of daily carrying a cross, or other such like instrument for this purpose.

Problem II.

TO MEASURE A LINE IMPEDED BY AN OBJECT NOT OBSTRUCTING THE SIGHT.



Let A B be a chain-line, the direct measurement of which is prevented by the unforeseen obstruction of the pond P. Measure A *n* till it reach to, or near to, the edge of the pond, as to *n*, and fasten the ends of the chain to the ground with arrows at *m* and *n*, the distance *m n* being made half a chain or 50 links. Take hold of the middle of the chain, and extend it firmly, till its two halves rest in the positions *mo*, *on*; thus making an equilateral triangle *m n o*, each side of which is 50 links. In the direction *m o*, measure to nearly opposite the middle of the pond, as to *q*. Again, make *p q* equal 50 links, fasten the ends of the chain at *p* and *q*, and extend its middle point to *r*, as before. In the direction *q r*, measure to *s*, till *q s* be equal to *m q*. Then *s* will be in the line A B, and *m s*† will be equal to *m q* or *q s*, which being added to A *m* will give the distance A *s*. Offsets being taken to the margin of the pond, during the measure-

* Since the parts B*n*, *nm* of the chain are together 80 links, of which B*n* is 30, the remainder *n m* is therefore 50; also B *m* was made 40; whence $40^2 + 30^2 = 50^2$, that is $m\text{B}^2 + \text{B}n^2 = mn^2$, therefore by Euc. I. 48, B*n* is perpendicular to B*m*, or *m*B*n* is a right angle.

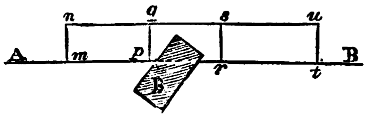
† Because the triangles *mno*, *pqr*, are both equilateral, the angles at *m* and *q* are each 60° or one-third a right angle; whence by Theorem IV. the angle at *s* is also 60°; therefore all angles of the triangle, and consequently its sides, are equal, that is, $m\,q = q\,s = m\,s$.

ment of the line *m q*, *q s*, and proper notes of the operation made in the field-book, the measurement from *s* to B may be continued.

Problem III.

TO MEASURE A LINE IMPEDED BY AN OBJECT OBSTRUCTING THE SIGHT, AS A BUILDING.

Let A B be a chain-line, the measurement of which is prevented by the building B. At *m*, four or five chains from the building, take a perpendicular *mn*, of such a length that the line *n s* may clear the building B. At or near the building take another perpendicular *pq*, exactly equal to *m n*, (these perpendiculars ought to be measured with the chain or a tape-line, if longer than the offset-staff,) and poles being put up, correctly vertical, at *n* and *q*, measure *q s u* in the direction *n q* of the poles, taking offsets to the building till it be cleared at *s*. Now on the line *qu*, at the distance *s u*, at or about equal to *m p*, erect the perpendiculars *s r*, *u t*, each exactly equal to *mn* or *pq*, fixing poles, correctly vertical, at *r* and *t*. These poles are evidently in the true direction A B, and the measurement of the line may now be continued from *r* to B, after adding the distance *q s* (which is equal to *p r*) to A *p*.



If the building, or other object, only protrude a few links over the line, the prependiculars *m n*, *p q*, *s r*, &c., may be erected by the offset-staff, as nearly correct as can be judged by the eye, and the results will be sufficiently accurate.

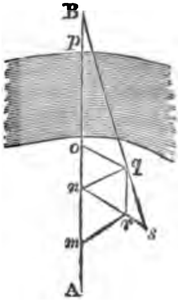
Note 1. When an object, as a pond or pit, not obstructing the sight, protrudes only a short distance over the line (see last figure); it will be sufficient to erect only the two equal perpendiculars *p q*, *r s* near its margin, with the offset-staff, as correctly as can be judged with the eye, and the distance *q s*, being measured, and added to A *p*, will give the distance A *r*.

Note 2. Some unskilful surveyors square off the line, as they call it, when it is obstructed by a building, or other object, that impedes the sight, in the following manner. On arriving at or near the obstruction, as at *p*, a perpendicular *p q* is erected to A *p*, another *q s* is erected to *p q*; a third *s r*, equal to *p q*, is erected to *q s*; and lastly a fourth perpendicular *r* B to *r s*. Though this last perpendicular is theoretically in the line A B, the student will at once perceive that when so many as four perpendiculars are taken, one upon another, at a very short distance from one another, that slight inaccuracies in the observations, as well as in the perpendicularity of the poles, placed at *p*, *q*, *s*, will have an almost unavoidable tendency to derange the accuracy of the work; since a small error, made at the beginning, multiplies as the operation proceeds. But by the method given in this Problem, the two perpendiculars on each side of the obstruction, are placed so far apart, that a slight deviation in the perpendicularity

of the poles cannot materially affect the accuracy of the work, while a slight error in erecting the perpendiculars, provided their lengths be made exactly equal, will not affect the work in the slightest degree. If, therefore, ordinary care be taken, the chance of error is almost impossible.

PROBLEM IV.

TO FIND THE WIDTH OF A RIVER, WHICH IS TOO WIDE TO BE REACHED ACROSS BY THE CHAIN.



Let A B be the chain line crossing a river, situated between o and p, a mark being fixed at p, on A B lay off $o\,n$, $n\,m$, each equal 50 links, and with the ends of the chain successively fixed at o, n, and at n, m, lay down the equilateral triangles $o\,q\,n$, $n\,r\,m$, as in Problem II., poles being fixed at n, q, and r. In the two directions $p\,q$, $n\,r$, fix a pole at s, and measure the distance rs accurately with a tape line to one-eighth of a link. Then by the similar triangles $s\,r\,q$, $q\,o\,p$, we shall have

$$rs : qr :: oq : op.$$

But $qr = oq = on = 50$ links, therefore,

$$rs : on :: on : op = \frac{on^2}{rs} = \frac{50^2}{rs} = \frac{2500}{rs}.$$

Whence the distance $o\,p$ becomes known.

For those who do not understand a rule, when symbolically expressed, we give, in words at length, the following.

RULE. Divide 2500 by the distance rs, and the quotient will be the breadth of the river, or the distance $o\,p$, which must be added to Ao to give the distance Ap.

EXAMPLES.

1. Required the breadth of a river by this method, when $r\,s$ measures 15 links.

$$\text{Here } \frac{2500}{15} = 166\tfrac{2}{3} \text{ links} = op.$$

2. When $r\,s$ measures $13\frac{1}{8}$ links, required the breadth of the river.

$$\text{Here } 2500 \div 13\tfrac{1}{8} = \frac{2500 \times 8}{105} = \frac{4000}{21} = 190\tfrac{1}{2} \text{ links nearly.}$$

PROBLEM V.

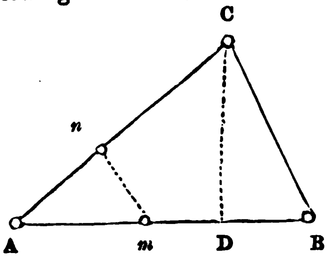
TRIANGULAR FIELDS.

When a triangular field, or piece of ground of that shape, is to be surveyed, set up poles or marks at each corner, and measure each side, leaving marks in at least two of the lines, and

entering their distances in the field-book; then measure the distance between the two marks for a proof-line:—or, one mark only may be left in one of the lines, which may be connected with its opposite angle for a proof-line.

EXAMPLES.

1. Required the construction and area of a field from the following dimensions.



	to ⊙ n	
Proof	384	line.
From	⊙ m	
	to ⊙ A	
	1244	
⊙ n	700	
	L. ⊙ C	
	to ⊙ C	
	852	
	L. ⊙ B	
	to ⊙ B	
	1338	
	1000	
⊙ m	600	
From	⊙ A	range E.

When the triangle A B C is constructed, the proof-line $m\,n$ will be found to measure 384 links, shewing that there has been no error in the work: also the perpendicular CD will be found to be 770 links; whence the area of the triangle $= 1338 \times 770 \div 2 = 5{\cdot}15130 = 5a.\ 0r.\ 24p.$ the area.

NOTE. If the proof line measured from the plan, does not exactly, or very nearly, agree with that measured in the field, some error has been made, and the work must be repeated.

TO FIND THE AREA OF A TRIANGLE FROM THE THREE SIDES.

RULE. From half the sum of the three sides subtract each side severally and reserve the three remainders; multiply the half sum continually by the three remainders, and the square root of the product will be the area.

NOTE. By this rule the area of a triangle may be found without laying it down, or finding the perpendicular.

Adopting the preceding example, we have by the rule,

$$\frac{1338 + 852 + 1244}{2} = 1717 = \text{half sum of the three sides.}$$

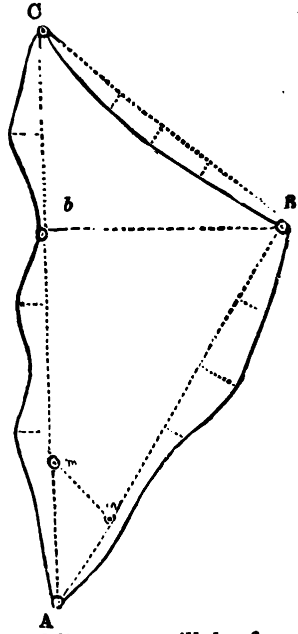
Then $1717 - 1338 = 379 =$ 1st remainder; $1717 - 852 = 865 =$ 2nd remainder; $1717 - 1244 = 473 =$ 3rd remainder; whence $\sqrt{(1717 \times 379 \times 865 \times 473)} = 5{\cdot}15992 = 5a.\ 0r.\ 25\tfrac{1}{2}p.$

the same as the area already found by measuring the perpendicular from the plan.

Note. This method of finding the areas of triangles is very little used in practice, on account of its requiring a tedious calculation, which may, however, be more readily performed by logarithms, as shall hereafter be shewn.

2. It is required to lay down a survey and find its content from the following field-notes.

	to ⊙ A	
	2504	0
	2000	74
⊙ *m*	1860	351 to ⊙ *n*
	1650	137
	1430	90
	1220	144
	850	30
	425	110
	000	0
	L. ⊙ C	
	to ⊙ C	
0	1346	
80	1072	
128	708	
98	458	
0	000	
	L. ⊙ B	
	to ⊙ B	
	1946	0
	1490	96
	1200	152
	1000	112
⊙ *n*	600	
	520	50
	000	
From	⊙ A	go N. E.



Having drawn the figure, the proof line *m n* will be found to measure 351 links, as in the field-notes; and the perpendicular B *b* to be 1056 links.

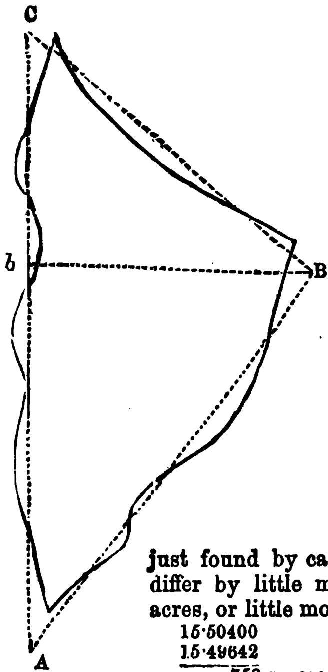
Double areas.

2644224	Triangle A B C
653112	Offsets on A B and A C
3297336	Sum
199016	Insets on B C
2)3098320	Difference

15·49160 = 15*a*. 1*r*. 38½*p*. nearly the area required.

COMPUTATION OF THE AREA BY CASTING, THAT IS, BY REDUCING THE CROOKED SIDES TO STRAIGHT ONES.

The offsets in the last example have been computed, in manner already shewn in Chap. II.; but by this method straight lines are drawn on the plotted figure so as to include as much space in the area to be measured, as they exclude, as nearly as can be judged by the eye, the area to be measured is thus reduced to a figure bounded by right lines only, which may thence be much more expeditiously reduced to triangles, trapeziums, &c. The method of drawing these lines is usually by a straight edged ruler of transparent horn, or by a silken thread stretched with a bow; the ruler or thread being moved over the crooked fence, till it appear to the eye to enclose as much of the adjoining ground as is left out, a line is then drawn in this position; and so on for other crooked fences. Thus the trouble of calculating numerous offsets is completely avoided, and with proper care equal accuracy is obtained.



2. We shall adopt the last example for this method of casting, that it may be seen how near the two methods agree.

The figure being constructed, and the boundary drawn carefully with ink, the chain-lines must then be rubbed out, and the three dotted lines A B, B C, C A must now be drawn, in such a manner, that the parts excluded by them may be equal to the parts included, as nearly as can be judged by the eye. The base A C will be found to be 2584 links, and the perpendicular B *b* = 1200 links. Whence

$$\frac{2584 \times 1200}{2} = 15{\cdot}50400 = 15a.\ 2r.\ 1p.$$

(nearly, *the area by this method.*)

If the area found by the true method be taken from the area, just found by casting, it will be seen that they differ by little more than one pole out of $15\frac{1}{2}$ acres, or little more than 1 in 4000: thus

$$\begin{array}{r} 15{\cdot}50400 \\ 15{\cdot}49642 \\ \hline 758 \end{array}$$ square links, or little more than one pole.

Note. It will hence be seen at once that a great deal of trouble is saved by this method, which is therefore generally adopted by practical surveyors; although it is certainly less correct than by calculating from the offsets, the former method depending chiefly on the accuracy of the casting lines for the truth of its results; but practice will soon render it easy to draw the lines so as to obtain almost perfect accuracy.

3. Required the plans and contents of two fields by both the methods of calculation, viz., offsets and by castings, from the following field-notes.

	To ⊙ B	
	1151	Proof line.
From	⊙ D	
	to ⊙ A	
	2640	
	1200	⊙ D
	R. ⊙ C	
	to ⊙ C	
0	1760	
63	1600	
145	1500	
190	1400	
117	1200	
120	1100	
189	1000	
120	760	
127	500	
120	400	
90	200	
111	000	
	R. ⊙ B	
	To fence.	
61	1861	
34	1750	⊙ B
95	1600	
129	1400	
150	1200	
130	1000	
115	800	
110	600	
60	400	
83	200	
0	000	
From	⊙ A	go E.

Content 18*a*. 3*r*. 33*p*.

	To ⊙ A	
0	2160	
18	2000	
99	1800	
220	1600	⊙ D to ⊙ B
259	1400	1198 proof line.
291	1245	
212	1100	
159	1000	
61	900	
32	820	
71	750	
109	600	
140	500	
147	400	
173	300	
140	200	
93	100	
0	000	
	R. ⊙ C	
	to ⊙ C	
0	1696	
61	1600	
119	1500	
161	1400	
140	1300	
96	1200	
40	1040	
91	900	
179	760	
0	660	
19	540	
140	400	
80	300	
98	200	
92	000	
	R. ⊙ B	
	To fence.	
	1598	
	1510	⊙ B
From	⊙ A	go E.

Content 17*a*. 2*r*. 27½*p*.

NOTE. It will be seen that in the main triangles of these two surveys, the proof lines have been taken from a side of each to its opposite angle; which is the best method of proof, when convenient to make it; but it may be performed with equal accuracy by taking a proof line from one side to another, at a short distance from one of the angles of the triangle.

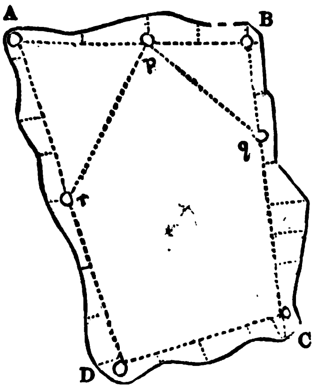
PROBLEM VI.

FOUR SIDED FIELDS.

When a field has four sides, straight or crooked, measure the four sides, or lines near them, if crooked, taking the offsets: also measure one or both the diagonals, one of which will serve as a base in plotting the work, and the other for a proof-line; or the proof-line may be measured in any other direction that may be most convenient.

Sometimes the measurement of both the diagonals is prevented by obstructions, in such cases it will be sufficient to measure tie-lines across two of the angles of the trapezium, at the distance of from two to five chains from each angle, according to the size of the field. These tie-lines with their distances from the angles on the main-lines will be found sufficient tor planning the lines and proving them.

EXAMPLES.



1. In the annexed figure the lines A B, B C, C D, D A are measured, marks being left at *p*, *q*, and *r*, and their respective distances on the lines noted in the field-book, thus furnishing the following method of laying down the plan.

On A B, as a base, take A *p* = given distance, and with the distances A *r*, *p r*, and centres A and *p* describe arcs cutting in *r*; then prolong A *r*, and lay off thereon the given length A D. In the same manner construct the triangle *p* B *q*, and make B C = its given length. Lastly, join D C, which must be of the length shewn in the field-book, otherwise there has been some mistake either in the measurement, or in laying it down. Should this be the case the whole of the work, firstly on the plan, and secondly in the field, must be gone over again till the error be discovered.

NOTE. When the main lines that include the chief part of the ground to be

measured are of considerable length, as from 30 to 40 chains, it will be necessary to take the tie-lines at least 10 chains from the angles, across which they are measured; for a small error, in laying down the plan with short tie-lines, will cause the main lines to deviate considerably from their true position when prolonged. However, it sometimes happens that long tie-lines cannot be obtained in consequence of obstructions. In such cases the tie-line must be carefully measured to even one-fourth of a link; the distance of each tie-line from its angle and the tie-line itself must then be all multiplied by 4, thus throwing fractions out of the question, and with the three lines, thus increased, the triangle determining the position of an angle of the trapezium, may be accurately constructed. The proof-line and its distances from its angle must be similarly treated, that the accuracy of the work may be fully established.

2. Required the plan and area of a straight-sided field from the following dimensions.

To ⊙ *n* 481 } Proof-line.. }	To ⊙ A 952 452 L. ⊙ D	⊙ *m*
To ⊙ B 833 } Proof-line.. }	to ⊙ D 1236 400 L. ⊙ C	⊙ *r*
	to ⊙ C 886 L. ⊙ B	
⊙ *n* From	to ⊙ B 1446 500 ⊙ A	go North.

When the figure has been laid down, the diagonal A C will be found = 1926, and the perpendiculars thereon from B and D respectively = 632 and 514 links. Whence the area is 11*a.* 0*r.* $5\frac{1}{4}p$.

3. Draw the plans, and find the contents of two enclosures, from the following field-notes, both by calculation from the offsets and by casting.

NOTE. In each of the two following examples, it will be seen that there are two straight sides, and two that require offsets: also, in the former example, one of the crooked sides is crossed by the chain-line, thus producing insets, the content corresponding to which must be subtracted, as in former cases.

	To ⊙ B	
	2921	diag.
	2000	
	1000	
Return	to ⊙ B	
	to ⊙ C	
	2960	diag.
	2000	
	1000	
	R. ⊙ A	
	to ⊙ A	
	1344	
	1000	
	R. ⊙ D	
	to ⊙ D	
0	2488	
93	2000	
140	1600	
171	1200	
135	800	
60	400	
0	000	
	R. ⊙ C	
	to ⊙ C	
0	1440	
171	1300	
223	1100	
	1000	
175	900	
Cross	721	fence
	600	165
	400	261
	200	199
	000	0
	R. ⊙ B	
	to ⊙ B	
	2700	
	2000	
	1000	
From	⊙ A	go W

Content 38*a.* 1*r.* 3½*p.*

	To ⊙ D	
	2478	diag.
	2000	
	1000	
Return	to ⊙ B	
	to ⊙ C	
	2652	diag.
	2000	
	1000	
	R. ⊙ A	
	to ⊙ A	
0	1030	
57	800	
99	600	
130	400	
106	200	
0	000	
	R. ⊙ D	
	to ⊙ D	
0	2750	
101	2600	
145	2400	
168	2000	
104	1600	
0	1310	
0	715	
88	400	
99	200	
0	000	
	R. ⊙ C	
	to ⊙ C	
	1328	
	1000	
	R. ○ B	
	to ⊙ B	
	1892	
	1000	
From	⊙ A	go E.

Content 28*a.* 0*r.* 19½*p.*

Problem VII.

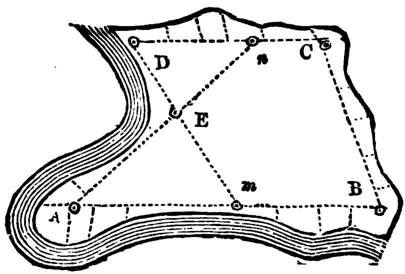
FIELDS HAVING MORE THAN FOUR SIDES.

Various methods will suggest themselves to the surveyor for taking lines to lay down a field that requires more than four main lines to take its boundary. The method of dividing such

fields into trapeziums and triangles is, in most cases, circuitous, and displays little skill on the part of the surveyor, especially where all the sides are crooked, and where a plan is required. A few methods of surveying fields of this kind will, therefore, be presented to direct the student; although their variety of shape is so endless, that no general rule can be given for laying out lines on the ground, that shall give an incontestably accurate plan. To tie every angle in succession, though true in principle, is by no means a safe method, especially where there are a great number of angles to be tied, as an error in one of the tie-lines will derange the whole of the work, without affording the means of detecting where the error lies.

NOTE. The following examples of surveys of this kind occurred in part of the author's extensive practice, as a surveyor of parishes, under the Tithe-Commissioners. The student is recommended to sketch the following specimens on a large scale, and find their contents by the usual methods.

EXAMPLES.



1. Here a field of five sides is surveyed by the same number of lines, viz. AB, BC, CD, D*m*, and A*n*, the last two intersecting in E. These lines evidently constitute a decisive proof among themselves and all of them are available in taking the boundary.

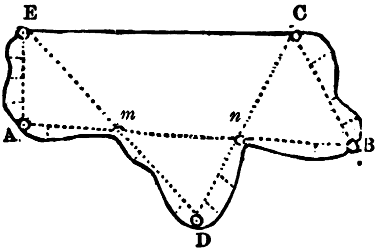
In surveying this field (poles or natural marks being supposed to be fixed at A, B, C, D, and E) commence close to the river's edge, in the line A B prolonged backwards, enter the offsets and the station A in the field-book. On arriving at ⊙ *m*, in the direction E D, enter its distance, and so on to ⊙ B, measuring the line to the fence; from B proceed to C, in like manner, measuring beyond the station to the fence. The place of the ⊙ *n* is to be noted, on arriving in the direction E A, while measuring C D. D *m* is next measured, the place of the ⊙ E being noted. Lastly, go from *m* to A, and measure A *n*, entering the place of the ⊙ E a second time, all the offsets being supposed to be taken during the operation.

Construction of the plan. Select the distances A *m*, A E, and E *m* from the field-book, and with them construct the triangle A *m* E, prolong the sides to their entire lengths, up to the boundaries, and fix the places of the stations B, *n*, and D.

Now, if the measured length of D *n* just fit between D and *n*, the work is right with respect to the triangles A E *m*, E D *n*. Lastly, prolong D*n* to the ⊙ C, and, if the distance from thence to the ⊙ B be the same as shewn by the field-book, the whole of the work is right. But, if the distance D *n* do not agree, the work must be examined from the beginning; if only the distance B C fail, then only that distance and the portions *m* B, C *n* need be examined.

NOTE. It may here be proper to add that, if a straight fence had passed at or near the stations *m* and *n*, excluding the portion of the field towards B and C, these stations would have determined the position of that fence, thus completing the survey of a five-sided field with only the four lines A *m*, *m* D, D *n*, *n* A, which may be measured consecutively.

2. The annexed figure A*m*D*n*BCE is a seven-sided field, or rather resolveable into a seven-sided field for the purposes of surveying; one of the sides CE is straight, and the fence A *m* D *n* B too much bent to be taken by one line, crossing and recrossing it, with offsets taken to the right and left. The lines here required are only five, viz. AB, BC, CD, DE, and E A, which may be measured consecutively.

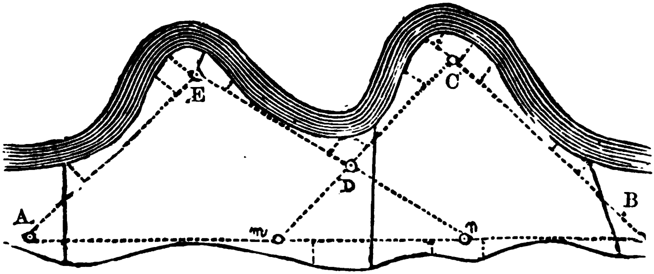


The stations *m* and *n* in A B, being carefully noted in the field-book, give at once the means of laying down the plan, and proving its accuracy. The student will at once perceive that the triangle *m n* D should be first laid down, and its sides D *m*, D *n*, prolonged to the stations C and E, from whence the lines C B, E A, must respectively reach the points B and C to confirm the accuracy of the work. This done, the offsets on the several lines may be laid off, through which the bent fences are to be drawn; the side, C E, being straight, is determined by joining the stations C and E, which are assumed to be therein, or, what means the same thing, by the edge of its ditch or drain.

NOTE. It ought here to be remarked, that by the ordinary method of surveying, this field would require nine lines to effect its survey, viz., for the trapezium A B C E, its four sides and two diagonals or tie-lines, thus making six lines; and for the triangle D*mn*, at least three other lines, (its tie-line included) thus making in all nine lines, or nearly twice the number required by the method here given.

3. The following figure comprises two fields, by the side of a river; each field, for the purposes of surveying, may be con-

sidered a five-sided field. The five lines A B, B C, C *m*, *n* E, and E A, are found amply sufficient to accomplish an accurate

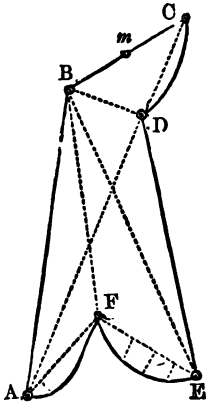


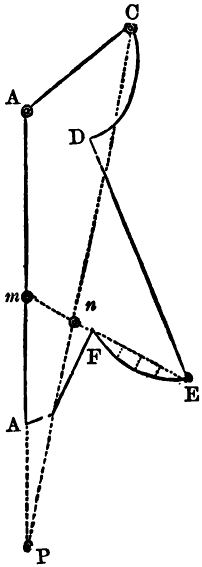
survey, in consequence of the three fences, terminating at the river being straight, their positions are determined by the intersections of the surveying lines, the middle fence by three intersections, and the two end fences by two intersections and one offset each, thus proving the accuracy of their positions. The two main triangles of the survey, viz. A*n*E, *m*B C, mutually prove one another by the intersection at the station D.

It will be unnecessary to explain the method of laying down the figure, as that will be obvious from what has been done before. It may, however, be proper to remind the student that the largest triangles ought to be laid down first, and the accuracy of the plan will be shewn by the agreement of the ⊙ D with the proper intersection of the lines C *m*, E *n*.

NOTE. By the common-place method, used by the generality of surveyors, the two fields, in the last example, are made to require no less than 12 lines instead of 5, as here shewn, viz., one in the same position as the A B, one near the three bends in the river and two close by the right and left straight fences; thus forming a trapezium, which, with two diagonals, or tie-lines, requires 6 lines in the usual way: the two triangles, abutting from the side of the trapezium next the river, requiring, as sides and tie-lines, three lines each, that is, 6 lines more for the purpose of taking the bends of the river; thus making 12 lines in all, as already stated.

4. Required the plan and the area of a field, having six sides, from the following field-notes, taken from an old, though still much practised, method. The first figure shews the lines according to the old, the second according to the improved method.





	To ⊙ B	
	1321	diag.
Return	to ⊙ F	
	to ⊙ E	
	1939	diag.
	L. ⊙ B	
	to ⊙ B	
	536	diag.
	L. ⊙ D	
	to ⊙ D	
	1664	diag.
	R. ⊙ A	
	to ⊙ A	
0	569	
69	400	
0	000	
	L. ⊙ F	
	to ⊙ F	
0	767	
81	600	
105	400	
65	200	
0	000	
	R. ⊙ E	
	to ⊙ D	1607 to ⊙ E, length of fence.
0	766	
6	400	
0	000	
	R. ⊙ C	
	to ⊙ C	
	950	⊙ m to ⊙ D 511 proof line.
	600	
	R. ⊙ B	
	to ⊙ B	
	1690	
	1000	
From	⊙ A	range North.

Area 13*a*. 1*r*. 7*p*.

It will be seen, from the field-notes and the first figure, no less than eleven lines are used to complete the survey by the old method; whereas four lines will be found amply sufficient to effect the same purpose: thus the three lines P B, B C, and CP, constituting the triangle PBC, are first measured, stations being left at *m* and *n* in the direction of the corner at E; the

line *m* E is then measured through the ⊙ *n*; thus proving the triangle PBC, and determing the position of the straight fence D E, which position is further proved by offsets and a crossing at, and south of D, the crooked fences having been taken by offsets in the usual way: thus completing the survey with little more than one third of the lines required by the former method. The student can readily add the field-notes in the latter case.

5. Required the plans and contents of two straight-fenced fields, each having five sides, from the following field notes, as given by old methods.

	To ⊙ A	
	2083	diag.
	1000	
Return	to ⊙ D	
	to ⊙ E	
	840	diag.
	R. ⊙ C	
From	to ⊙ C	to ⊙ *m* 519
	1537	diag.
	1000	
	R. ⊙ A	
	to ⊙ A	
	1170	
	R. ⊙ E	
	to ⊙ E	
	1035	
	R. ⊙ D	
	to ⊙ D	
	730	
	L. ⊙ C	
	to ⊙ C	
	779	
	R. ⊙ B	
	to ⊙ B	
	2107	
	2000	
	1200	⊙ *m*.
Begin	at ⊙ A	go W

Content 12*a*. 2*r*. 5*p*.

	To ⊙ B	
	755	diag.
	R. ⊙ E	
	to ⊙ E	
	619	diag.
From ⊙ *m*	200	to ⊙ D 142
	R. ⊙ C	
	to ⊙ C	
	599	diag.
	R. ⊙ A	
	to ⊙ A	
	196	
	R. ⊙ E	
	to ⊙ E	
	346	
	R. ⊙ D	
	to ⊙ D	
	309	
	R. ⊙ C	
	to ⊙ C	
	267	
	R. ⊙ B	
	to ⊙ B	
	867	
Begin	at ⊙ A	range W.

Content 1*a*. 2*r*. 36*p*.

NOTE. In the two preceding examples, nine lines are required in each case, by the circuitous methods there adopted. Each field, by the improved methods, here laid down, may be surveyed by five lines, or both fields by only one line

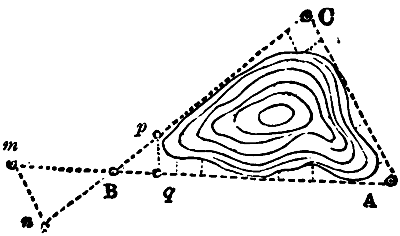
more than are required by the methods given in the preceding field notes. Precisely the same system of lines may be adopted in the former case, as has been done in the Example 1. The latter case is left to the ingenuity of the student.

Problem VIII.

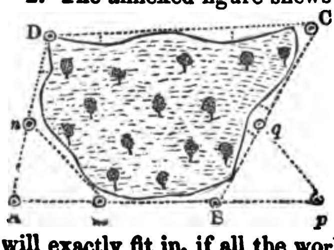
WOODS, LAKES, AND SWAMPY GROUNDS.

When woods, lakes, and swampy grounds are required to be surveyed, where lines cannot be measured upon them, a system of lines must be adopted for each particular case, so combined by triangulation as to prove their accuracy, among themselves, when laid down on paper. If the wood, or other inaccessible space, (as far as measuring is concerned) be either of, or very near, a triangular shape, the three sides of a triangle will compass it, which may be proved by a tie-line at one of its corners, if the wood or lake will admit one of sufficient length, but, if not, any two of the sides of the triangle may be prolonged for this purpose, offsets, or rather insets, being taken to the boundary of the wood or lake, in the usual way.

1. Here the three sides of the triangle A B C compass a lake or large pond, insets being taken therefrom to the margin of the water. The accuracy of the work is proved by the tie-line *p q*, or, if this line be thought too short, (as it apparently is) the sides AB, CB may be prolonged to *m* and *n*, till B *m*, B *n*, each equal about one third of AB, and the tie-line *mn*, being measured, will establish the accuracy of the work, or prove it wrong, as the case may be.



2. The annexed figure shews the survey of a wood, which is effected by the four main-lines A B, B C, C D, D A, the first line being prolonged to *p*, and stations being left at *m*, *n*, and *q* for the tie-lines *m n*, *p q*. It scarcely need be added that when the lines A B, A D, B C, are laid down by the help of the two tie-lines, the line C D will exactly fit in, if all the work has been accurately done.



3. Required the plan and area of a wood from the following field notes, two of the fences of which are straight.

	To ⊙ *p*	
	1175	
⊙ C 0	825	
28	600	
0	500	⊙ *n* 491
	400	to ⊙ *m*
0	275	
20	150	
52	25	
0	000	
Return	to ⊙ B	
	to ⊙ *m*	
	900	
⊙ B 0	600	
25	500	
0	425	
0	300	
50	200	
40	125	
0	000	
From	⊙ A	go E.

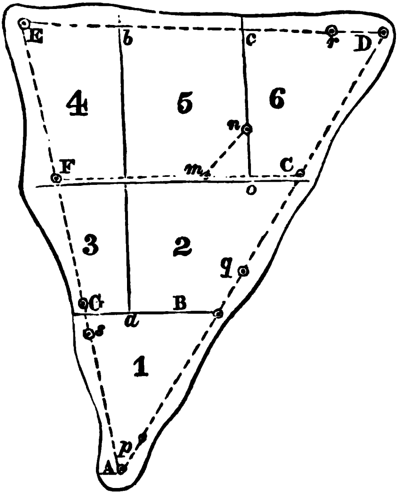
	To ⊙ A	
0	812	
	⊙ D	
	to ⊙ D	
	575	⊙ *q* 550
	400	to ⊙ *p*
Return	to ⊙ C	

The figure being laid down, and reduced by casting the two crooked sides, the area will be found 458,065 square links, or 4*a*. 2*r*. 13*p*.

NOTE. As a general rule for surveying woods and lakes, the following may be given:—Measure as many lines round the boundary to be surveyed as will compass it, and tie all the angles, except the two last, as in the preceding examples; you will thus obtain a system of lines that will prove among themselves, as the last measured side will just reach from the last station to the first, if the work have been done with accuracy. But the shapes of woods, &c., are so various that it would not be adviseable, in every case, to adhere to this method: much must, therefore, be left to the skill of the surveyor. In the southern counties of England, where coppice wood is sold by the acre for fuel, it is very frequently required to survey a portion of a wood, (the coppice being cut down, and the large timber still standing); in such cases, the lines must be taken within the space to be surveyed, as the adjoining uncut coppice prevents their being taken outside. The lines must, therefore, be ranged among the growing timber, as well as they can; and the tie-lines taken through the most convenient openings left by the trees. Surveys of this kind, it may be proper to add, are best performed with the help of the Prismatic Compass, or the Box Sextant, which are sufficiently accurate for these surveys, which are always of small extent.

TO SURVEY A SMALL ESTATE, DIVIDED INTERNALLY BY STRAIGHT FENCES.

A small estate, of a form nearly triangular, is divided into six fields by four straight fences, as shewn on the annexed plan. The survey is commenced at A, by measuring the line A D, taking the offsets, and leaving stations at the crossing of these fences at B and C. The line D E is now measured, also crossing two of the straight fences. From the ⊙ E the line E A is measured, leaving stations at the hedge-crossings F and G: thus completing the triangle A D E. Next, the line BG is measured, close to the straight fence B G, and crossing the straight fence *a b*. Lastly, the line F C is measured, again crossing the straight fences *a b* and *c o*: a station being left at *m*, about a chain's distance from *o*, and one chain measured close by the fence *oc* to *n*, and from thence to ⊙ *m*: (the last operation being, in most cases, the most readily performed before the line passes the fence *c o*) thus completing the survey. The position of the straight fence *a b* is proved to be correct by the crossing of three of the lines of the survey, viz., B G, F C, and D E; the crossings made by these three lines on *a b*, must be in a right line on the plan, otherwise there has been an error in the field notes. The straight fence *o c* has only two crossings by the surveying lines FC, DE, its position is, therefore, not duly proved to be correct without the tie-line *m n*, measured in the manner already stated. This expeditious method of proving the position of a straight fence, crossed by two chain lines (as the fence *c o*), was never adopted till done so by the author, in the parish survey under the *Tithe Commissioners, who, in their instructions to surveyors, directed the lengths of all such fences to be measured to establish the accu-*



racy of their positions. Now the measurement of the length of such a straight fence as *c o*, which is nearly perpendicular to the two chain lines that cross it, gives a very imperfect proof of the accuracy of its position, especially if it be a long one. In this latter case the fence *c o* might be a full chain from its true position at one end, and its length, as shown by the plan. would be so near the measured length that the error would not be detected; whereas the tie line *m n*, shown on the plan, would detect the error at once, by its increased or diminished length. However, when two chain lines cross a straight fence obliquely, the length of the fence, from crossing to crossing, evidently gives a sufficient proof of the accuracy of its position; yet to measure a short tie line, at one of the crossings, is a shorter method of proof, in all cases where the length of the fence is considerable. The method of proof recommended by the Commissioners in question, ought, therefore, never to be adopted, except where one or both of the chain lines cross the straight fence obliquely and at a short distance. It seems hardly necessary to remark that the positions of the straight fences G B, C F are determined by the lines measured close to them. *Thus the survey of six fields may be made, and its accuracy proved, by five lines, with the two short tie lines m n, n o, which may be regarded as mere offsets. Besides, had the fences* G B, C F *been crooked, the same lines would have effected the survey by offsets thereon. Moreover, the survey of these six fields, all the internal fences being as shown in the plan, may be accurately effected by four lines, in the following manner.* The triangle A D E remaining as the foundation of the survey, let a station, or rather direction point, *p* be entered in the field notes, in the direction of the straight fence *a b*, and another similar point at *q*, in the direction of the straight fence *o c*; leave also a proper station mark, or pole, at *r*; on arriving at *s* in E A, leave another station, in such a position, that a line from *s* to *r* will cross all the four straight fences. This last line will prove the fundamental triangle ADE and the positions of the four straight fences, at the same time, without measuring the lines by the fences G B, C F; which can have now three crossings by chain lines, and the other two fences *a b*, *o c* have each two crossings by chain lines, and each one direction point, viz: *p* and *q*; through which points these fences must respectively pass, after they have been drawn through their crossings on D E, and the other line from *s* to *r*, not shown on the plan. This method of determining the position of straight fences, though theoretically elegant, cannot always be easily

practised, especially where the fences are high, or the ground hilly, thus preventing the directions of the straight fences being seen to distant chain lines, as in the cases of the fence *a b, o c,* with respect to the chain line A D.

THE METHOD OF MEASURING HILLY GROUND.

When the ground, over which lines are measured, rises or falls, or both alternately, the horizontal distances are what are required in plotting the survey, as well as for finding the content thereof, and not the actual distances measured along the surface of the ground.

For many ordinary purposes the horizontal measurement may be obtained by holding the end of the chain up so as to keep it horizontal, as nearly as can be judged by the eye, the arrow being placed vertical under the end so held up: but when a large and accurate survey is required, the distances must be measured along the line of ground, and the angles of elevation and of depression of the several inclined parts of the line must be taken, either with a common quadrant, or afterwards with the theodolite (to be hereafter described), and the lengths of the several rises and falls must be noted; from which the corresponding correct horizontal distances may be readily computed. The following table shews the number of links to be subtracted from every chain, or 100 links, for the angles there set down.

TABLE shewing the reduction in links and decimals of a link upon 100 links for every half degree of inclination from 3° to 20° 30′.

Angle.		Reduction.	Angle.		Reduction.	Angle.		Reduction.
3°	0′	0·15	9°	0′	1·23	15°	0′	3·41
	30	0·19		30	1·37		30	3·64
4	0	0·24	10	0	1·53	16	0	3·87
	30	0·31		30	1·67		30	4·12
5	0	0·38	11	0	1·84	17	0	4·37
	30	0·46		30	2·01		30	4·63
6	0	0·55	12	0	2·19	18	0	4·89
	30	0·64		30	2·37		30	5·17
7	0	0·75	13	0	2·56	19	0	5·45
	30	0·86		30	2·76		30	5·74
8	0	0·97	14	0	2·97	20	0	6·03
	30	1·10		30	3·19		30	6·33

By this table the trouble of computation is avoided, only the

distance, measured on each rise or fall, requiring to be multiplied by the reduction in chains corresponding to the angle of each rise or fall, and the product, subtracted from that distance, will give the correct distance, as in the following

EXAMPLES.

1. A line was measured 12·43 chains, on ground having a rise of $8\frac{1}{2}$ degrees, required the horizontal length of the line.

Here to $8\frac{1}{2}°$, or 8° 30′ corresponds the reduction 1·10 links, whence

12·43	Whence 12·43
1·10	13·673
13·6730 links	12·29·327 horizontal distance,

in which the decimal, being less than half a link, is rejected; thus making the correct horizontal distance 12·29 chains, or 1229 links.

2. The acclivity of a hill rises 20°, and measures 16·14 chains, its declivity falls $11\frac{1}{2}°$ and measures 32·28 chains, required the horizontal distance between the extremities of the line thus measured over the hill, it being level at the top 2·80 chains,

Here $16·14 \times 6·03 = 97·3242$
$32·28 \times 2·01 = 64·8828$

162·2070 or 162 links, or 1·62 chains.

Whence 16·14
32·28
2·80

51·22
1·62

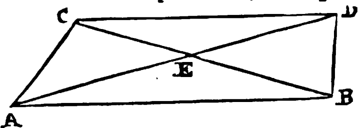
49·60 chains, the horizontal distance required.

NOTE. When fences are crossed, stations made, &c., on the acclivity or declivity of a hill, the horizontal distance up to such points must be found. Some surveyors place the arrow forward a distance equal to reduction, due to the angle of acclivity or declivity, at the end of every chain measured, and thus obviate the necessity of reducing the line afterwards, having for the purpose a small pocket quadrant, so graduated that the plumb-line thereof shews, on observing the angle of elevation or depression, the reduction required for each chain. A great deal of trouble is thus saved as the theodolite cannot be conveniently carried about for this purpose. Such pocket quadrants are not made by the mathematical instrument makers, being the productions of clock makers or other mechanics, according to the various designs of surveyors.

THE USE OF THE PARALLEL RULER,

IN REDUCING CROOKED FENCES TO STRAIGHT ONES, TO FACILITATE THE COMPUTATION OF THE CONTENTS OF FIELDS.

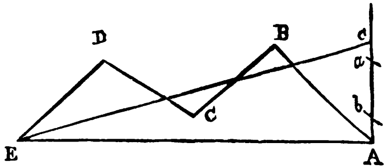
As some surveyors prefer the parallel ruler to the method already given, for reducing crooked fences to straight ones, the method of using that instrument for this purpose is here given. This method is founded on a well-known proposition of Euclid, in which it is shewn that *triangles on the same base, and between the same parallels, are equal.*



Let A B C, A B D be triangles on the same base AB, and between the same parallels A B, C D; then the triangle ABC is equal to the triangle A B D. And, if the triangle A E B, which is common to the other two triangles, be taken away, the remaining triangles A E C, B E D will also be equal; whence equal areas may be transferred from one side of a line to the other, which is the principle, on which, as already said, the following Problems are founded.

PROBLEM I.

IT IS REQUIRED TO REDUCE THE OFFSET-PIECE A B C D E TO A RIGHT ANGLED TRIANGLE A E *c*, BY AN EQUALIZING LINE E *c*, WITH THE PARALLEL RULER.



Draw the indefinite line A *c* perpendicular to A E. Lay the parallel ruler from A to C; hold the near side of the ruler firmly, and move the further side to B, which will cut A *c* at *a*, where a mark must be made. Lay the ruler from *a* to D, and the further side thereof being now held fast, bring the near side to C, marking A *c* at *b*. Lay the ruler from *b* to E, move it parallel to D, marking A *c* at *c*. Join E *c*; then A E *c* is a right angled triangle required, and its area may be found by taking half the product of A E and A *c*.

THE FOLLOWING IS A GENERAL RULE FOR SOLVING PROBLEMS OF THIS KIND.

Draw a temporary line, as A *c*, at right angles, or at any other angle to the chain line, as A E, of the offsets.

1. Lay the ruler from the first to the third angle, and move it parallel to the second angle; then make the first mark on the temporary line.

2. Lay the ruler from the first mark on the temporary line to the fourth angle, and move it parallel to the third angle; then make the second mark on the temporary line.

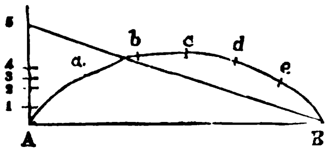
3. Lay the ruler from the last named mark to the fifth angle, and move it parallel to the fourth angle; then make the third mark on the temporary line.

4. Lay the ruler from the last named third mark on the temporary line to the sixth angle, and move it parallel to the fifth angle; then make the fourth mark on the temporary line.

In this manner the work of casting by the parallel ruler may be conducted to any number of angles. Great care must be taken, during the operation, to prevent the ruler slipping, as such an accident will derange the whole of the work, if not discovered and immediately corrected.

Problem II.

TO REDUCE A CURVED OFFSET-PIECE TO A RIGHT-ANGLED TRIANGLE.



Let A *a b c d e* B be the curved offset-piece. Divide the curve by points *a*, *b*, &c., so that the parts A *a*, *a b*, &c., may be straight or nearly so; and draw A 5 perpendicular to A B. Lay the ruler from A to *b*; move it parallel to *a*, and mark A 5 at 1. Lay the ruler from 1 to *c*; move it parallel to *b*, and mark A 5 at 2. Lay the ruler from 2 to *d*; move it parallel to *c*, and mark A5 at 3 Lay the ruler from 3 to *e*; move it parallel to *d*, and mark A 5 at 4. Lay the ruler from 4 to B; move it parallel to *e*, and mark A5 at 5. Draw the line B5; then will A B 5 be a right angled triangle equal in area to the offset-piece A *a b c d e* B, as required.

EXAMPLES FOR PRACTICE ON THE TWO PRECEDING PROBLEMS.

1. Lay down a right-lined offset-piece, from the following notes; reduce it to a triangle by the parallel ruler; and find its content, both by calculation from the offsets and the casting of the ruler.

	To ⊙ B	
	751	0
	550	150
	400	51
	250	99
	50	75
	000	0
From	⊙ A	range West.

The area found by calculation from the offsets is 0*a.* 2*r.* 17*p.* and the perpendicular of the triangle, found by the casting of the parallel ruler, is 161 links. Hence $\frac{751 \times 161}{2} = 0.60455$ square links, or somewhat more than 0*a.* 2*r.* 16½*p.* It hence appears that the method of casting by the ruler, gives the content very near the truth. In fact this method is mathematically accurate; but the danger of error, as already said, arises from the accidental slipping of the ruler during the operation.

2. Lay down and find the area of a curve-lined offset-piece, from the following field notes: and find its area both by calculation, and by the casting of the parallel ruler.

	to ⊙ B	
0	600	
95	500	
131	400	
134	300	
126	200	
89	100	
0	000	
From	⊙ A	range East.

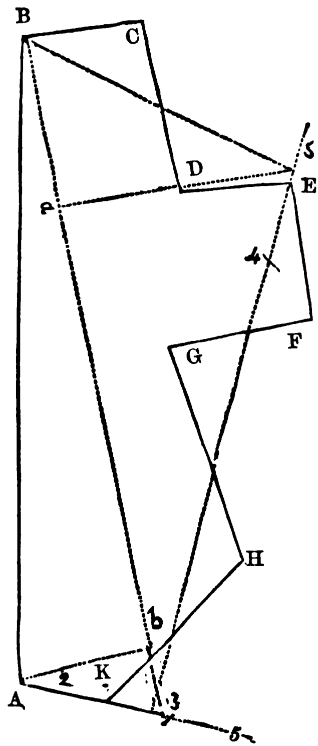
By both the methods the area is found to be 0*a.* 2*r.* 15½*p.* nearly, the perpendicular on A B being found 199 links by the casting operation.

Problem III.

TO REDUCE THE IRREGULAR FIELD A B C D E F G H K TO A TRAPEZIUM BY THE PARALLEL RULER.

Prolong the line A K at pleasure. Lay the ruler from K to G; move it parallel to H, and mark A K prolonged at 1. Lay the ruler from 1 to F; move it parallel to G, and mark A K

at 2. Lay the ruler from 2 to E; move it parallel to F, and mark A1 at 3. Draw a line from 3 to E and prolong from E. Lay the ruler from E to C; move it parallel to D, and mark 3 E at 4. Lay the ruler from 4 to B; move it parallel to C, and mark 3 E prolonged at 5. Draw a line from 5 to B; then shall A B 5 3 be a trapezium, equal in area to the irregular figure A B C D E F G K; the area of which may be found by multiplying the diagonal B 3 by half the sum of the perpendiculars thereon from A and 5.

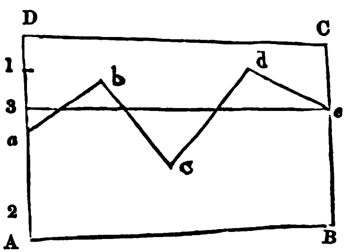


NOTE. In this manner the crooked sides of a field may be successively reduced to straight ones. Thus, if the side A B had been crooked, the operation of straightening might be continued by prolonging the dotted line 5 B, and find successive points therein, corresponding to the assumed angles, till the last angle was brought thereon, and so on with respect to the side A K, had it also been crooked. When the sides of a field are curved, the method of reducing them to straight lines is the same as shewn in Problem II.

PROBLEM IV.

TO DRAW AN EQUALIZING LINE THROUGH THE CROOKED FENCE *a b c d e*, SO THAT THE TWO FIELDS A B *e a*, *a* D C *e* MAY BE FOUR SIDED.

Lay the ruler from *a* to C; move it parallel to *b*, and mark A D at 1. Lay the ruler from 1 to *d*; move it parallel to *c*, and mark A D at 2. Lay the ruler from 2 to *e*; move it parallel to *d*, and mark A D at 3. Draw the line *e* 3, and it will divide the two fields, so that their quantities shall be the same as those before separated by the crooked fence *a b c d e*.



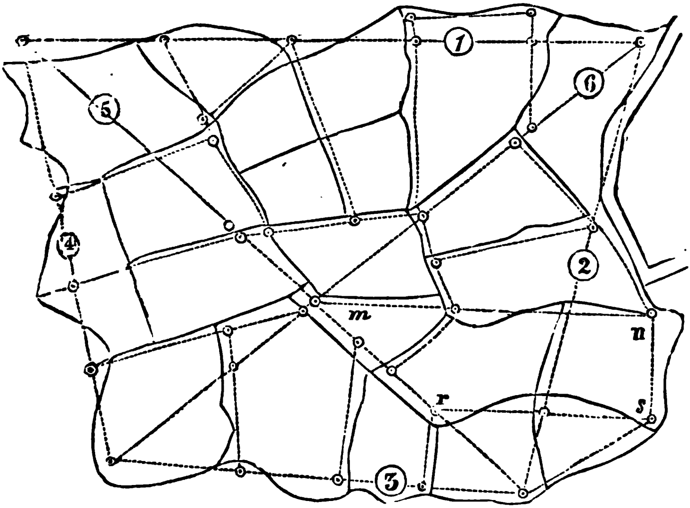
It is scarcely necessary to add, that had the fence *a b c d e* been curved, the equalizing line might have been found as in Prob. II.

NOTE. The owners of adjoining estates sometimes agree to straighten fences or boundaries, by an equalizing line of fence, or as they term it, "by giving and taking equal quantities of land." When this is required to be done, the crooked or curved boundary fence must be first measured on the ground, and then plotted on a large scale; when the equalizing line may be drawn on the plan, as shewn in the last Problem, and the distance from D to 3, or from *a* to 3, must be correctly measured by the scale: this distance must next be measured in the field, accordingly as it is taken from D or *a*; and the new line of boundary *e* 3 may now be ranged and marked out, preparatory to making the required fence. Moreover, to guard against errors, in the preceding operation, it is advisable to measure, both on the plan and on the ground, the parts cut off, on each side of the new boundary line, thus proving the work, as an error is of serious consequence in these matters: and, if one is found to exist, it must be corrected before the boundary fence is made.

SURVEYING LARGE ESTATES OR PARISHES BY THE CHAIN ONLY.

1. Having perambulated the boundary of the estate, parish, or lordship to be surveyed, if you find that its boundary approaches somewhat near to that of a four-sided figure, or trapezium, the system of fundamental lines, adopted by order of the Tithe Commissioners of England and Wales, is to be preferred. These fundamental lines are six in number, of which four must run close by, or as nearly as possible to, the boundary in question, thus forming a trapezium, four lofty station poles being placed at each angle, as objects for running the lines; the other two lines must form the diagonals of this trapezium, and therefore pass through the central parts of the survey, intersecting each other, the points of intersection being noted on measuring each line, so that when the system of lines are laid down on the plan, the proof of the accuracy of the work may be fully established, before the minor operations, or filling up, as it is called, is commenced. It will be necessary, moreover, in almost every case, to range the lines between every two of the main stations with long slender ranging poles, as the intervention of hills, fences, trees, buildings, &c., will frequently interrupt the view of even the loftiest station poles that can be obtained; and more especially so, when the main stations are at a great distance, which depends on the magnitude of the survey, and is sometimes as much as ten miles. In measuring these main-lines, every fence, road, stream, building, &c.. which is passed or crossed must be noted in the

field-book, the several crossings and bends being sketched therein, to the latter of which offsets must be taken. Stations must also be left on these main-lines, at convenient situations for taking the interior fences, &c., of the survey, and their distances carefully noted in the field-book. From and to the stations, thus left, or from and to points near them, secondary lines must be run, as near the interior parts of the survey as possible, the crossings, offsets, and other remarks being made in the field-book, as already directed for the measurement of the main-lines. These secondary lines will accurately fit between the points from and to which they have been measured, when laid down on the plan; thus forming a net work of small triangles within the four large triangles, into which the survey is divided by the six fundamental lines. This principle of proof is founded on the obvious property of triangles having a common angle always fitting one within the another, the common angle of both being coincident. The lines marked with the figures 1 to 6, represent the system in question, those without figures are the secondary lines.

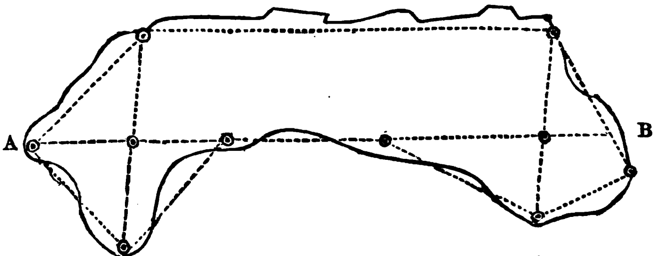


The main lines are numbered with the figures 1, 2, 3, &c., in small circles, as the most convenient method of reference to the field-book: the secondary lines must have these numbers continued on them, for the same purpose, but this is not done in the diagram, to avoid confusing it.

It will be seen that the secondary lines *m n*, *r s* are prolonged beyond the system of main lines to give stability to the parts of the survey that protrudes beyond line 2. It will also be seen that the positions of several straight fences are determined by triple intersections of the main and secondary lines, or by direction points taken in them, as shewn in Prob. IX.

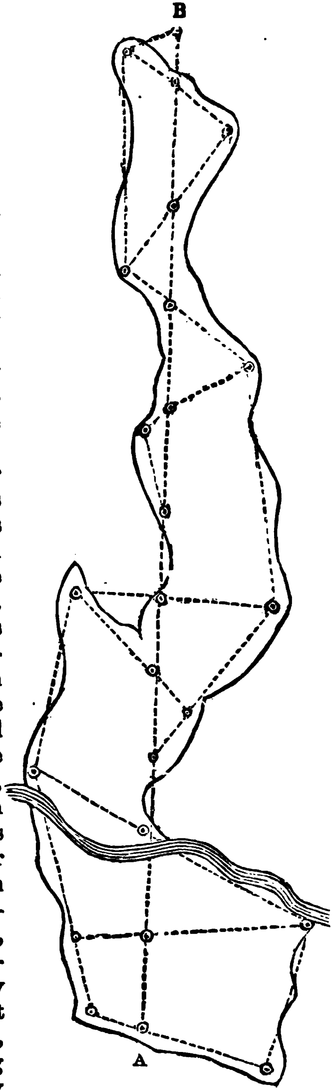
In the author's practice, as a surveyor, he adopted this method, in the survey of the Parish of Tillington, Sussex, with the exception of bringing one of the diagonals to one of the angles of the trapezium, which, of course terminated in one of the side lines; thus giving a system of lines equally perfect, the end of the diagonal being only a few chains from the angle; there being a considerable practical difficulty in laying down this system of lines, where the ground is hilly and woody, as was the case on this occasion.

2. The system of main lines adopted by the author in the survey of the Parish of Woolbedding, in the same county, was the following,



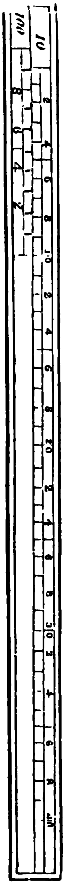
the base line A B being about five miles in length. To this survey, it may be clearly seen that the Tithe Commissioners' system of lines would not be at all adapted; though some surveyors, in compliance with their orders, adopted their system, whatever might be the shape of the survey, thus wasting much valuable time, in running lines over grounds at a great distance, in some parts, from the parish to be surveyed, and incurring the ridicule of both the scientific and the ignorant to boot. Such a course would evidently be required, in using their system in this survey: meanwhile a system of lines adapted to the shape of the survey, and constituting proof among themselves, as those shewn above do, are evidently the best. In this figure the interior fences and secondary lines of the survey are not drawn, as their great number would confuse the student; the author's object being to present a proper system of fundamental lines for the survey in question

3. The Parish of Lodsworth, in the same county, presents another variety of shape; in the survey of which, the author at first laid the system of fundamental lines shewn in the annexed figure; which had been adopted, had he not, in the meanwhile, obtained the survey of the adjoining parish; both parishes were, therefore, included in one survey. The base-line A B of this parish was nearly seven miles in length, its long, narrow, zig-zag shape completely setting aside the *universal* method put forth by the Tithe Commission authorities; who, however, did not insist on their methods being adhered to, as they approved of the author's maps as of the first class, in this and many other cases where their method was not adopted. But such was the obsequiousness or ignorance of the great majority of surveyors that, even in such incongruous cases as the one referred to, they persevered in the Tithe Office rule, in some cases by joining together two, three, or four trapeziums, with their diagonals, and sometimes by making the surveys after their own methods, and then drawing on their maps, the system, or groups of the systems, in question, and making a field-book to correspond thereto; they were thus at liberty to project lines in any direction they chose, without the trouble of measuring them; and many have exultingly confessed they did so, after their maps had received the seal of



the Tithe Commissioners: but how far such behaviour is to be commended, I leave the reader to judge.

Having given the student a great variety of examples, both in small and large surveys, it will now be proper to describe the drawing instruments, required in laying down extensive surveys, as well as the more perfect field instruments, required in railway and other engineering surveys; which are often of a complicated character, as are also private surveys of wood-lands and mountainous districts. These subjects will be treated of in Chap. V., where the proper forms of the field books will be given, for extensive surveys of every description.



CHAPTER IV.

DRAWING & SURVEYING INSTRUMENTS.*

THE VERNIER SCALE.

The nature of this scale will be best understood from its construction.—To construct a vernier scale, which will enable us to take off a number to three places of figures: divide all the primary divisions into tenths, and number these subdivisions, 1, 2, 3, &c., from the left hand towards the right, throughout the whole extent of the scale. Then take off, with the compasses, eleven of these subdivisions, set the extent off backwards, from the end of the first primary division, and it will reach beyond the beginning of this division, or zero point, a distance equal to one of the subdivisions. Now divide the extent, thus set off, into ten equal parts, marking the divisions on the opposite side of the divided line to the strokes marking the primary divisions and the subdivisions; and number them 1, 2, 3, &c., backwards from right to left. Then, since the extent of eleven subdivisions has been divided into ten equal parts, so that these ten parts exceed by one subdivision the extent of ten subdivisions, each one of these equal parts, or, as it may be called, one division of the vernier scale, exceeds one of the subdivisions by a tenth part of a subdivision, or a hundreth part of a primary division. In our figure the distances between the primary divisions are each one inch, consequently the distances between the subdivisions are each one-tenth of an inch, and the distances of the divisions on the vernier scale are each one tenth and one-hundredth of an inch.

* See also: "Mathematical Instruments," by J. F. Heather, Weale's Series, 1s. 6d.

TO TAKE OFF A GIVEN NUMBER FROM THE VERNIER SCALE.

Example 1. Let the given number be 253.

Increase the first figure 2 by 1, making it 3; because the vernier scale commences at the end of the first primary division, and the primary divisions are measured from this point, and not from the zero point.* The first figure, thus increased, represents 35 of the subdivisions from the zero point, from which the third figure 3 must be subtracted, leaving 32, since three divisions of the vernier scale will contain three of these subdivisions together with three-tenths of a subdivision. Now place one point of the compasses upon the third division of the vernier scale, and extend the other point to the 32nd subdivision, or the second division beyond the third primary division, and, laying down the distance between the points of the compasses, it will represent 253, or 25·3, or 2·53, according as the primary divisions are taken for hundreds, tens, or units.

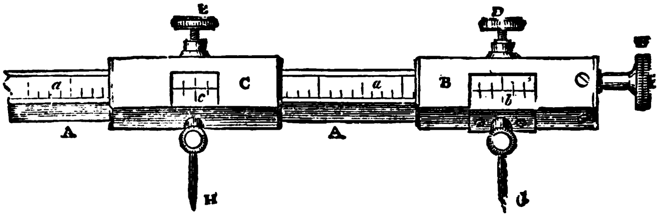
General Rule.—*To take any number to three places of figures from this vernier scale.*—Increase the first figure by one; snbtract the third figure from the second, borrowing one from the first increased figure, if necessary, and extend the compasses from the division ugon the vernier scale indicated by the third figure, to the subdivision indicated by the number remaining after performing the above subtraction.

DIRECTIONS CONCERNING PLOTTING EXTENSIVE SURVEYS FROM THIS SCALE.

During the time necessarily occupied in plotting an extensive survey, the paper on which the work is drawn, is affected by the different states of humidity of the air, and the parts laid down from the same scale, at different times, will not exactly correspond, unless the scale has been first laid down on the paper itself, and all the distances of the survey have been taken from the scale so laid down, which will always be in the same state of expansion or contraction as the work on the plan. Therefore, for plotting an extensive survey, and accurately filling in the minutiæ, a diagonal, or vernier scale may be advantageously laid upon the paper on which the plan is to be made. A vernier scale is preferable to a diagonal scale, because in the latter it is extremely difficult to draw the diagonals with accuracy, and we have no check upon its errors; while in the former, the uniform manner in which the strokes of one scale separate from those of the other, is strong evidence of the truth of both.

* If the vernier scale were placed to the left of the zero point, a distance less than one primary division could rot always be found upon the scale.

THE BEAM COMPASSES.



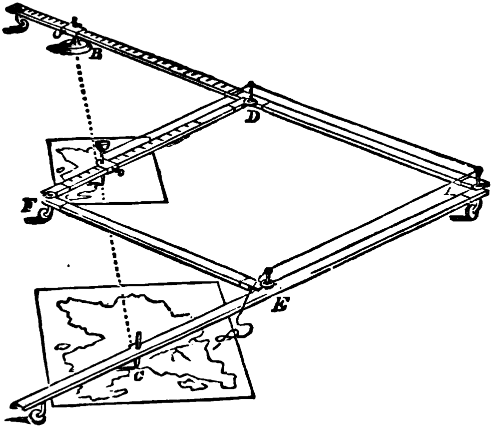
The above engraving represents the instrument, which consists of a beam A A, of any length required, generally made of well-seasoned mahogany. Upon its face is inlaid throughout its whole length, a slip of holly or boxwood *a a*, on which are engraved the divisions or scale, either in feet and decimals or inches and decimals, or such parts of either as may be required. Those made for the use of the Ordnance survey of Ireland, were divided to a scale of chains, 80 of which occupied a length of six inches, which, therefore, represented one mile, six inches to the mile being the scale on which that important survey is plotted; the present survey of the metropolis being plotted to a scale of 60 inches to a mile.* Two brass boxes B and C are adapted to the beam; of which the latter may be moved, by sliding to any part of its length, and its position fixed by tightening the clamp screw E. Connected with the brass boxes are the points of the instrument G and H, which may be made to have any extent of opening by sliding the box C along the beam, the other box B being firmly fixed at one extremity of the beam. The object to be attained by this instrument is the nice adjustment of the points G, H to any definite distance apart. This is accomplished by two verniers, or reading places *b*, *c* each fixed at the side of an opening in the brass boxes, to which they are attached, and affording the means of minutely subdividing the principal divisions *a a* on the beam, which appear through these openings. D is a clamp screw for a similar purpose to the screw E, that is, to fix the box B, and prevent motion in the point it carries after adjustment. F is a slow motion screw, by which the point G may be moved any minute quantity for perfecting the setting of the instrument.

* The first-class parish maps of the Tithe Commission survey are to a scale of three chains to one inch, or one mile to 26⅔ inches. Plans of private estates are to various scales, but chiefly varying from 5 to 12 chains to an inch, the smaller scales being chiefly adopted where the estates are large, that the plans may be of a convenient size. The Ordnance survey of most of the towns in Ireland is also plotted on a scale of 120 inches to the mile, the others being plotted on a scale of 60 inches to the mile.

The method of setting the instrument may be understood from the above description of its parts, in conjunction with the following explanation of the method of examining and correcting the adjustment of the vernier *b*, which like all other mechanical adjustments, will occasionally become deranged. This verification must be performed by means of a detached scale. Suppose, for example, that the beam compass is divided into feet, inches, and tenths, and subdivided by the vernier into hundredths, &c. First set the zero division of the vernier to the zero of the principal divisions on the beam, by means of the slow motion screw F. This must be done with great care. Then slide the box C, with its point H, till the zero on the vernier C exactly coincides with any principal division on the beam, as 12 or 6 inches. To enable us to do this with extreme accuracy, some superior kinds of beam compasses have the box C also furnished with a tangent or slow motion screw, by which the setting of the points of division may be performed with the utmost precision. Lastly, apply the points to a similar detached scale, and, if the adjustment be perfect, the interval of the points G, H will measure on it the distance to which they were set on the beam. If they do not by ever so small a quantity, the adjustment should be corrected, by turning the screw F, till the points exactly measure that on the detached scale; then, by loosening the little screws which hold the vernier *b* in its place, the position of the vernier may be gradually changed, till its zero coincides with the zero on the beam; and, the screws being now tightened, the adjustment will be complete.

THE PANTAGRAPH.

The pantagraph, used for reducing or enlarging maps, consists of four brass bars A B, A C, D F, and F E: the two longer bars A B, A C are connected by a moveable joint at A: the two shorter bars are connected in like manner with each other at F, and with the longer bars at D and E, and, being equal in length to the portions A D, A E of the longer bars, form with them an accurate parallelogram A D F E, in every position of the instrument. Several ivory castors support the machine parallel to the paper, and allow it to move freely over it in all directions. The arms A B and D F are graduated and marked $\frac{1}{2}$, $\frac{1}{3}$, &c., and have each a sliding index which can be fixed to any of the divisions by a milled-headed clamping screw, seen in the engraving. The sliding indices have each of them a tube, adapted either to slide on a pin, rising from a heavy circular weight called the fulcrum, or to receive a sliding holder with a pencil or pen, or a blunt tracing point, as may be required.



When the instrument is correctly set, the tracing point, pencil, and fulcrum will be in one straight line, as shewn by the dotted line in the figure. The motions of the tracing point and pencil are then each compounded of two circular motions, one about the fulcrum, and the other about the joints at the ends of the bars, upon which they are respectively placed. The radii of these motions form sides about equal angles of two similar triangles, of which the dotted right line B C, passing through the tracing point, pencil, and fulcrum, forms one side: hence the distances passed over by the tracing point and pencil, in consequence of either of these motions, have the same ratio, and, therefore, the distances passed over in consequence of the combination of the two motions, have also the same ratio, which is that indicated by the setting of the instrument.

The engraving represents the pantagraph in the act of reducing a map to the scale of half the original. For this purpose the sliding indices are first clamped at the divisions on the arms, marked $\frac{1}{2}$; the tracing point is then fixed in the socket at C, over the original map; the pencil is next placed in the tube of the sliding index, on the bar D F, over the paper to receive the copy; and the fulcrum to that at B, on the bar A B. The machine being now ready for use, if the tracing point C be passed delicately and steadily over every line of the map, a true copy, but of one half of the scale of the original, will be marked by the pencil on the paper beneath it. The fine thread represented as passing from the pencil quite round the instrument to the tracing point C, enables the draughtsman at the tracing point to raise the pencil from the paper while he passes the

tracer from one part of the original to another, and thus to prevent false lines being made on the copy. The pencil holder is surmounted by a cup, into which sand or shot may be put, to press the pencil more heavily on the paper, when found necessary.

If the object were to enlarge the map to double its scale, then the tracer must be placed on the arm D F, and the pencil at C; and, if a copy were required of the same scale as the original, then, the sliding indices still remaining at the same divisions on D F and A B, the fulcrum must take the middle station, and the pencil and tracing point those on the exterior bars A B, A C of the instrument.

Though the pantagraph affords the most rapid means of reducing a map or drawing, we cannot recommend its use for enlarging a copy, or even for copying on the same scale, especially when the original drawing is a complicated one. The eidograph described in Heather's "Surveying and Astronomical Instruments," in Weale's Series, should be employed for that purpose, or one or other of the following methods.

To produce a copy of the same size as the original.—Lay a sheet of tracing paper, having its under side rubbed over with powdered black lead, upon the paper intended to receive the copy; the original being then placed over both, the whole may be made to lay steadily by weights placed thereon: the tracing point may now be carefully passed over all the lines of the drawing, with a pressure proportionate to the thickness of the paper; and the paper beneath will receive corresponding marks, forming an exact copy, which is afterwards to be inked in.

NOTE. Copies of maps are also taken for ordinary purposes by laying tracing paper thereon, through which, from its almost transparent thinness, all the lines of the original can be seen, and readily traced with ink, on the tracing paper. Copies, thus obtained, are called tracings.

Another method.—The drawing or map is placed on a large sheet of plate-glass, called a copying glass, and the paper to receive the copy placed over the drawing. The glass is then fixed in such a position as to have a strong light to fall upon it from behind, and to shine through it and both the original drawing and the paper to receive the copy. By this means the lines of the original drawing become visible through the paper to receive the copy, which can be made with precision and ease, without any risk of soiling or injuring the original.

To copy with exactness on a reduced or enlarged scale.—For this purpose we have recourse to the method of squares, by which the most minute details may be copied with accuracy. This perhaps may be best shown by an example. Let figure 1, in the annexed engraving represent a plan of an estate, which

it is required to copy upon a reduced scale of one half. The copy will therefore be half the length and half the breadth; and consequently will occupy but one fourth the space of the original. The subject is the map of an estate, but the process would be precisely the same, if it were an architectural, mechanical, or any other drawing.

Fig. 1.

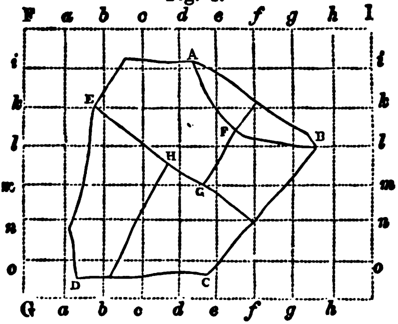
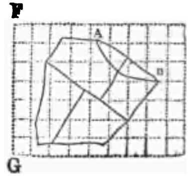


Fig. 2.



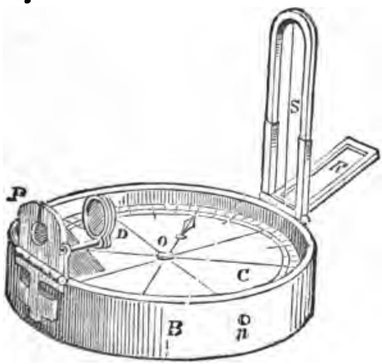
Draw the lines F I, F G at right angles to each other; from the point F towards I and G, set off any number of equal parts, as F *a*, *a b*, *b c*, &c., on the line F I, and F *i*, *i k*, *k l*, &c., on the line F G: from the points in the line F I, draw lines parallel to the other line F G, as *a a*, *b b*, *c c*, &c., and from the points on F G, draw lines parallel to F I, as *i i*, *k k*, *l l*, &c., which being sufficiently extended towards I and G, the whole of the original drawing will be covered with a reticule of small but equal squares. Next draw upon the paper intended for the copy, a similar set of squares, but having each side only one-half the length of the former, as is represented in figure 2. It will now be evident that if the lines AB, BC, CD, &c., figure 1, be drawn in the corresponding squares in figure 2, a correct copy of the original will be produced, and of half the original scale. Commencing then at A, observe where in the original the angle A falls, which is towards the bottom of the square, marked *d e*. In the corresponding square, therefore, of the copy, and in the same proportion towards the left-hand side of it, place the same point in the copy: from thence tracing where the curved line A F crosses the bottom line of that square, which crossing is about two-fifths of the width of the square from the left hand corner towards the right, and cross it similarly in the copy. Again, as it crosses the right hand bottom corner in the second square below *d e*, describe it so in the

copy; find the position of the points similarly where it crosses the lines *ff* and *gg*, above the line *ll*, by comparing the distances of such crossings from the nearest corner of a square in the original, and similarly marking the required crossings on the corresponding lines on the copy. Lastly, determine the place of the point B, in the third square below *gh* on the top line; and a line drawn from A in the copy, through these several points to B, will be a correct reduced copy of the original line. Proceed in like manner with every other line on the plan, and its various details, and you will have the plot or drawing laid down to a small scale, yet bearing all the proportions in itself exactly as the original.

It may appear almost superfluous to remark, that the process of enlarging drawings, by means of squares, is a similar operation to the above, excepting that the points are to be determined on the smaller squares of the original, and transferred to the larger squares of the copy. The process of enlarging, under any circumstances, does not, however, admit of the same accuracy as reducing.

THE PRISMATIC COMPASS.

With this instrument horizontal angles can be observed with great rapidity, and, when used with a tripod stand, with a considerable degree of accuracy; it is, therefore, a useful instrument for filling in the details of an extensive survey, after the principal points have been laid down by means of observations made with the theodolite, hereafter to be described. It was used for this purpose by the gentlemen engaged in making the Ordnance surveys.



C is a compass card, divided usually to every 20′, or third

part of a degree, and having attached to its under side a magnetic needle; *n* is a spring, which, being touched by the finger, acts upon the card and checks its vibrations, so as to bring it sooner to rest, when making an observation. S is the sight vane, having a fine thread stretched along its opening, which is to cut the point to be observed by the instrument. The sight vane is mounted upon a hinge joint, so that it can be turned down flat in the box, when not in use. P is the prism, attached to a plate sliding in a socket, and thus admitting of being raised or lowered at pleasure, and also supplied with a hinge joint, so that it can also be turned down into the box, when not in use. In the plate to which the prism is attached, and which projects beyond the prism, is a narrow slit, forming the sight through which the vision is directed, when making an observation. On looking through the slit, and raising or lowering the prism in its socket, distinct vision of the divisions on the compass card, immediately under the sight-vane, is soon obtained; and these divisions, seen through the prism, all appear, as each is successively brought into coincidence with the thread of the sight-vane by turning the instrument round, as continuations of the thread, which is seen distinctly through the part of the slit that projects beyond the prism.

The method of using the instrument is as follows:—the sight-vane S, and the prism P, being turned up on their hinge joints, as represented in the figure, hold the instrument as nearly in a horizontal position as you can judge, or, if a tripod stand be used, set it as nearly as you can in a horizontal position by moving the legs of the stand, that thence the card may play freely. Raise the prism in its socket till the divisions on the card are seen distinctly through it, and, turning the instrument round, until the object to be observed is seen through the portion of the slit projecting beyond the prism, in exact coincidence with the thread of the sight-vane, bring the card to rest by touching the spring *n*; and then reading at the division upon the card, which appears in coincidence with the prolongation of the thread, gives the magnetic azimuth or bearing of the object observed, or the angle which a straight line, drawn from the eye to the object, makes with the magnetic meridian.* The magnetic azimuth of a second object being

* The magnetic meridian now makes an angle of 22° with the true meridian at London, the north point of the compass being 22° west of the true north point. This angle is called the variation of the compass, and is different at different places, and also at the same place at different times. Since this variation will affect equally, or nearly so, all azimuths observed within a limited extent, and during a limited time, the angles subtended by any two of the

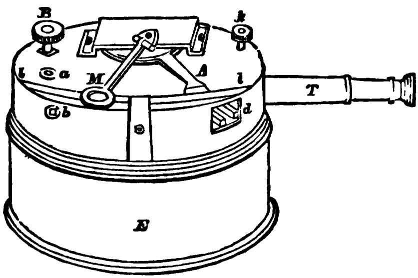
obtained in the same manner, the difference between these two azimuths is the angle subtended by the objects at the place of the eye, and is quite independent of the error in the azimuths, arising from the slit in the prism not being diametrically opposite to the thread of the sight-vane.

For the purpose of taking the bearings of objects much above or below the level of the observer, a mirror R is supplied with the instrument, which slides on and off the sight-vane S, with sufficient friction to remain at any part of the vane that may be desired. It can be put with its face either upwards or downwards, so as to reflect the images of objects considerably above or below the horizontal plane of the eye of the observer. If the instrument be used for obtaining the magnetic azimuth of the sun, the dark glasses D must be interposed between the sun's image and the eye.

There is a stop in the side of the box, not shewn in the figure, by touching which a little lever is raised and the card thrown off its centre; as it always should be, when not in use, or the constant playing of the needle would wear the fine agate point, on which it is balanced, and the sensibility of the instrument would be thereby impaired. The sight-vane and prism being turned down, a cover fits on the box, which is about three inches diameter, and one deep; and the whole being packed in a leather-case, may be carried in the pocket without inconvenience.

THE BOX SEXTANT.

This instrument, which is equally portable with the prismatic compass, forming, when shut up, a box about three inches in diameter, and an inch and a half deep, will measure the



objects observed, being the difference of their azimuths, will not be affected by the variation; and hence the map or plan may be constructed with all the objects in their proper relative positions; but the true meridian must be first laid down on the map, if required, by making allowance for the variation.

actual angle between any two objects to a single minute. It requires no support but the hand, is easily adjusted, and, when once adjusted, seldom requires re-adjusting.

When the Sextant is to be used, the lid E of the box, is taken off and screwed to the bottom, where it makes a convenient handle for holding the instrument; the telescope T, being then drawn out, the instrument appears as shewn in the figure A is an index arm, having at its extremity a vernier, of which 30 divisions coincide with 29 divisions on the graduated limb *l l*, and the divided spaces on the limb denote each 30 minutes, or half a degree, the angles observed being read off by means of the vernier to a single minute. The index is moved by turning the milled head B, which acts upon a rack and pinion within the box. To the index arm is attached a mirror, called the index glass, which moves with the index arm, and is firmly fixed upon it by the maker, so as to have its plane accurately perpendicular to the plane in which the motion of the index arm takes place, and which is called the plane of the instrument; this plane is evidently the same as the plane of the face of the instrument, or of the graduated limb *l l*. In the line of sight of the telescope is placed a second glass, called the horizon glass, having half its surface silvered, and which must be adjusted that its plane may be perpendicular to the plane of the instrument, and parallel to the plane of the index glass, when the index is at zero. The instrument is provided with two dark glasses, which can be raised or lowered by the little levers seen at *d*, so as to be interposed, when necessary, between the mirrors and any object too bright to be otherwise conveniently observed, as objects in the direction of the sun. The eye end of the telescope is also furnished with a dark glass, to be used when necessary.

To see if the instrument be in perfect adjustment.—Place the dark glass before the eye-end of the telescope, and looking at the sun, and moving the index backwards and forwards a little distance on either side of zero, the sun's reflected image will be seen to pass over the disc, as seen directly through the horizon glass, and if in its passage the reflected image completely covers the direct image, so that one perfect orb is seen, the horizon glass is perpendicular to the plane of the instrument: but, if not, the screw at *a* must be turned by the key *k* till such is the case. The key *k* fits the square heads of both the screws seen at *a* and *b*, and fits into a spare part of the face of the instrument, so as to be at hand when wanted. This adjustment being perfected, bring the reflected image of the sun's

lower limb in exact contact with the direct image of his upper limb, and note the reading of the vernier; then move the index back beyond the zero division of the limb, till the reflected image of the sun's upper limb is in exact contact with the direct image of his lower limb, and, if the zero of the vernier be now exactly as far behind the zero of the limb, as it was at the former reading in front of it, the instrument is in perfect adjustment; but, if not, half the difference is the amount of error, which must be corrected by applying the key *k* to the screw at *b*, and turning it gently till both readings are alike, each being made equal to half the sum of the two readings first obtained. When this adjustment is perfected, if the zeros of the vernier and limb are also made exactly to coincide, the reflected and direct image of the sun will exactly coincide, so as to form but one perfect orb, and the reflected and direct image of any line, sufficiently distant not to be affected by parallax, as the distant horizon, or the top or end of a wall, more than a half a mile distant, will coincide so as to form one unbroken line.

To obtain the angle subtended by two objects in, or nearly in, the same horizontal plane.—Hold the sextant in the left hand, and bring the reflected image of the right hand object into coincidence with the direct image of the left hand object, and the reading of the instrument will give the angle between the two objects.

To obtain the angle subtended by two objects in, or nearly in, the same vertical plane.—Hold the instrument in the right hand, and bring down the reflected image of the upper object by turning the milled head B, till it exactly coincides with the direct image of the lower object, and the reading of the instrument will give the angle between the two objects.

It will be seldom that the surveyor need pay any attention to the small error arising from parallax, but, should great accuracy be desirable, and one of the objects be distant while the other is near, the parallax will be eliminated by observing the distant object by reflection, and the near one by direct vision, holding the instrument for this purpose with its face downwards, if the distant object be on the left hand. If both objects be near, the reflected image of a distant object, in a direct line with one of the objects, must be brought into coincidence with the direct image of the other object, and the parallax will thus be eliminated.

For the purposes of surveying, the horizontal angles between objects are chiefly required, and the reduction of these angles from the actual oblique angles subtended by the objects, would be a troublesome process. If the angle subtended by two objects be large, and one be not much higher than the other,

the actual angle observed will, however, be a sufficiently near approximation to the horizontal angle required; and if the angle between the two objects be small, the horizontal angle may be obtained, with sufficient accuracy, by taking the difference of the angles observed between each of the objects, and a third object at a considerable angular distance from them. With a little practice the eye will be able to select an object in the same direction as one of the objects, and nearly on a level with the other object, and the angle between this object and the object selected will be the horizontal angle required.

For laying off long offsets, or perpendicular distances from a station line.—The pocket sextant is a most convenient instrument for this purpose: for by setting the index to 90°, and walking along the station line, looking through the horizon glass directly at the further station staff, or any other remarkable object on the station line, any object off the station line will be seen by reflection, when the observer arrives at the point where the perpendicular falls from this object upon the station line, and the distance from this point to the object, being measured, is its perpendicular distance from the station line.

THE OPTICAL SQUARE.

For the purpose of measuring long offsets and perpendiculars, this instrument is now very generally used, which consists of the two glasses of the sextant fixed permanently at an angle of 45°, so that any two objects seen in it, the one by direct vision and the other by reflection, subtend at the place of the observer an angle of 90°.

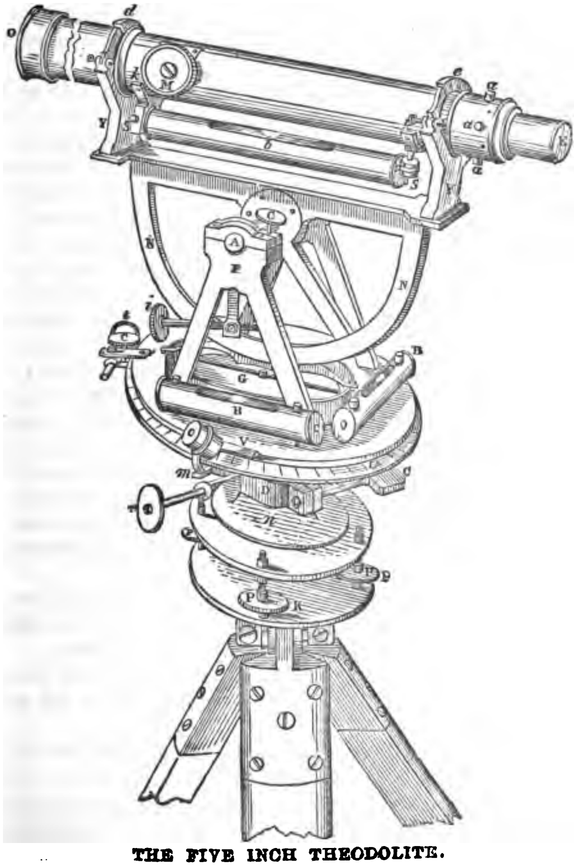
THE THEODOLITE.

The theodolite is the most important instrument used by surveyors, and measures at the same time both the horizontal angles between two objects observed with it, and the angles of elevation of these points from the point of observation.

The instrument may be considered to consist of three parts; the vertical limb for measuring vertical angles, the horizontal limb for measuring horizontal angles, and the parallel plates, in the lower of which is a female screw, adapted to the staff-head, which is connected by brass joints with three mahogany legs, so constructed as to shut together and form one round staff, a very convenient form for portability, and, when opened out, to make a firm stand, be the ground ever so uneven.

The horizontal limb is composed of two circular plates L and V, which fit accurately one upon the other. The lower plate projects beyond the other, and its projecting edge is sloped off, and graduated at every half-degree. The upper plate is called the vernier plate, and has portions of its edge sloped off, so as

to form with the sloped edge of the lower plate continued portions of the same conical surface. These sloped portions of the upper plate are graduated to form the verniers, by which the limb is subdivided to minutes. The five-inch theodolite, represented in the figure, has two such verniers 180° apart. The lower plate of the horizontal limb is attached to a conical axis passing through the upper parallel plate, and terminating in a ball fitting in a socket, upon the lower parallel plate. This axis is hollowed to receive a similar conical axis, ground accurately to fit it, so that the axis of the two cones may be exactly coincident. To the internal axis, the upper, or vernier



THE FIVE INCH THEODOLITE.

plate of the horizontal limb is attached, and thus, while the whole limb can be moved through any horizontal angle required, the upper plate only can also be moved through any desired angle, when the lower plate is fixed by means of the clamping screw C, which tightens the collar D. T is a slow-motion screw, which moves the whole limb through a small space, to adjust it more perfectly, after tightening the collar D by the clamping screw C. There is also a clamping screw *c* for fixing the upper plate to the lower, and a tangent screw *t*, for giving the upper plate a slow motion upon the lower, when so clamped. Two spirit levels B, B are placed upon the horizontal limb, at right angles to each other, and a compass G is also placed upon it, in the centre between the supports F F of the vertical limb.

The vertical limb N, N is graduated on one side at every 30 minutes, each way from 0 to 90°, and subdivided by the vernier, which is fixed to the compass box, to single minutes. Upon the other side are engraved the number of links to be deducted from each chain, for various angles of inclination, in order to reduce distances measured on ground rising or falling at these angles, to the corresponding horizontal distances. The axis A of this limb must rest in a position truly parallel to the horizontal limb, upon the supports F F, so as to be horizontal when the horizontal limb is set truly level, and the plane of the limb N N must now be perpendicular to its axis. On the top of the vertical limb N N is attached a bar that carries two Ys (so called from their shape), for supporting the telescope, which is secured by two clips *c*, *d*; and underneath the telescope is a spirit level *s s*, attached to it at one end by a joint, and at the other by a capstan-headed screw. The horizontal axis A can be fixed by a clamping screw C; and the vertical limb can then be moved through a small space by the slow-motion screw *i*.

Before commencing observations with this instrument, the following adjustments must be attended to:—

1. *Adjustments of the telescope for parallax and collimation.*

2. *Adjustments of the horizontal limb for setting the levels on the horizontal limb to indicate the verticality of its axis.*

3. *Adjustment of the vertical limb for setting the level beneath the telescope to indicate the horizontality of the line of collimation.*

1. *Parallax and collimation.* Move the object-glass *o* by the screw M, and the eye-glass E with the hand, till distant objects and the cross wires within the telescope, appear clearly defined; and the adjustment for parallax will be completed. Next, direct the telescope to some well-defined object at a

great distance; and see that the intersection of the cross wires cut it accurately; then loose the clips *c*, *d*, that confine the telescope in the Ys, and turn it round on its axis, observing whether the centre of the wires still continue to cut the object, during the whole revolution. If it does, it is in adjustment; if not, the line of collimation, or optical axis of the instrument, is not in the line joining the centres of the eye and object-glasses. *To correct this error*, turn the telescope on its axis, and by means of the four conjugate screws *a*, *a*, &c., that move the cross wires, correct for half the error, alternately loosing one screw and tightening its opposite one, till the cross wires cut the same point of the distant object, during an entire revolution of the telescope round its axis.

2. *Adjustment of the horizontal limb.*—Set the instrument up as level as you can by the eye, by moving the legs of the stand. Tighten the collar D by the clamping screw C, and, unclamping the vernier plate, turn it round till the telescope is directly over two of the parallel plate screws. Bring the bubble *b* of the level *s s*, beneath the telescope, to the centre of its run, by turning the tangent screw *i*. Turn the vernier plate half round, bringing the telescope again over the same pair of parallel plate screws; and, if the bubble of the level be not still in the centre of its run, bring it back to the centre, half-way by turning the parallel plate screws, over which it is placed, and half-way by turning the tangent screw *i*. Repeat this operation till the bubble remains accurately in the centre of its run, in both positions of the telescope; and, then turning the vernier plate round till the telescope is over the other pair of parallel plate screws, bring the bubble again to the centre of its run by these screws. The bubble will now retain its position while the vernier plate is turned completely round, shewing that the internal axis, about which it turns, is completely vertical. The bubbles of the levels on the vernier plate being now, therefore, brought to the centres of their tubes, will be adjusted, and also shew the axis to be vertical. Now, having clamped the vernier plate, loosen the collar D by turning back the screw C, and move the instrument slowly round on the external axis, and, if the bubble of the level *s s* maintain its position during a complete revolution, the external and internal axes are coincident, both being vertical at the same time; but, if the bubble does not maintain its position, it shews that the two parts of the axis have been inaccurately fitted, and the fault can only be remedied by the instrument-maker.

3. *Adjustment of the vertical limb.*—The bubble of the level

s s being in the centre of its tube, reverse the telescope end for end in the Ys, and, if the bubble does not remain in the same position, correct for one half of the error by means of the capstan-headed screw at the end of the level, and for the other half by the vertical tangent screw *i*. Repeat the operation till the result is perfectly satisfactory. Next turn the telescope round a little both to the right and to the left, and, if the bubble does not remain in the centre of its run, the level *s s* must be adjusted laterally by the screw at the other end. This adjustment will probably disturb the first, and the whole operation must be carefully repeated. By means of the small screw, fastening the vernier of the vertical limb to the vernier plate over the compass-box, set the zero of the vernier to the zero of the limb, and the vertical limb will be in perfect adjustment.

NOTE. The adjustments of the theodolite here given are essentially the same as those given by Heather, in his 'Treatise on Mathematical Instruments,' as being adapted to the plate of the instrument, which is taken from his work, though I prefer the concise methods, I have given at page 318 of my Additions to the ninth edition of 'Nesbit's Surveying.'

TO TAKE A HORIZONTAL ANGLE WITH THE THEODOLITE.

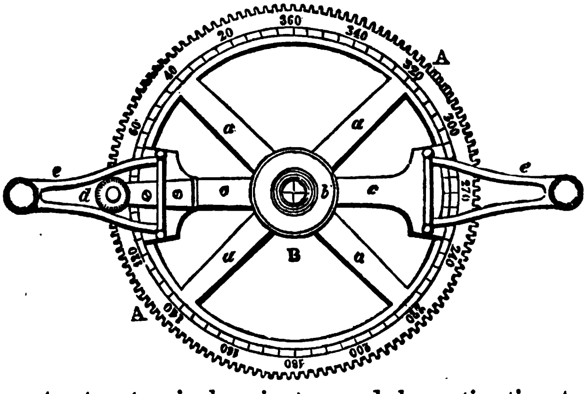
The theodolite being assumed to be in proper adjustment, the bubbles in the two levels B B, by a proper opening of the legs of the instrument should be made nearly central, and the plummet suspended beneath it should also hang over the station at which the angle is to be taken: then unclamp the whole instrument by means of the screw C, keeping the other motions clamped, and set the horizontal limb level, as already shewn in the first adjustment. Now clamp the whole instrument and unclamp the vernier plate; set the arrow of the vernier to 360°, or zero, on the lower plate, adjusting the points carefully by the microscope *m*, and the adjusting screw *i*. Again unclamp the whole instrument, turning it to the left of the two stations, between which the angle is to be taken, till the centre of the cross wires in the telescope cut the pole, flag, or other object in the station; then clamp the screw C, and by gently turning the screw T, the most perfect accuracy may be secured. Next unclamp the vernier plate and turn it round till the cross wires cut the object at the second station; then clamp and adjust the vernier plate, as before, and, having obtained perfect accuracy, read off the angle by means of the vernier with the microscope *m*. Lastly, read off the angle, in the same way, with the other vernier, and the mean, or half sum, of the two angles will be the correct angle.

TO TAKE A VERTICAL ANGLE.

Having set the instrument level, as already explained, observe at the same time whether the zero of the vertical limb coincides with that of its vernier, by the microscope attached thereto. These points being found coincident, raise or depress the telescope, till its optical axis, or cross wires, cut the object required; then clamp, and adjust till perfect accuracy be obtained, when the angle may be read off, which will be an angle of depression, if the arrow of the vernier be between the zero of the vertical circle and the object glass of the telescope; otherwise an angle of elevation.

THE CIRCULAR PROTRACTOR.

This instrument is a complete circle A A, connected with its centre by four radii *a, a, a, a.* The centre is left open and surrounded by a concentric ring, or collar, *b*, which carries two radial bars *c c*. At the extremity of one bar is a pinion *d*, working in a toothed rack quite round the outer circumference of the protractor. To the opposite extremity of the other bar is fixed a vernier, which subdivides the primary divisions on



the protractor to single minutes, and by estimation to 30 seconds. This vernier, as may be readily seen from the engraving, is carried round the protractor by turning the pinion *d*. Upon each radial bar *c, c* is placed a branch *e, e*, each branch carrying at its extremity a fine steel pricker, whose points are kept above the surface of the paper by springs placed under their supports, which give way when the branches are pressed downwards, and allow the points to make the necessary puncture on the paper. The branches *e, e* are attached

to the bars *c*, *c* with a joint, which admits of their being folded backwards over the instrument, when not in use, and for packing in its case. The centre of the instrument is represented by the intersection of two lines, drawn at right angles to each other, on a piece of plate glass, which enables the person using it to place it so that the centre, or intersection of the cross lines, may coincide with any given point on the plan. If the instrument is in correct order, a line connecting the fine pricking points with each other would pass through the centre of the instrument, as shewn by the intersection of the cross lines on the glass; which it may be observed, are drawn so nearly level with the under surface of the instrument as to do away with any serious amount of parallax, when setting the instrument over a point, from which any regular lines are to be drawn. In using this protractor the vernier should first be set to zero, or the division marked 360, on the divided limb, and then placed on the paper, so that the fine steel points may be on the given line, from whence the angular lines are to be drawn, and that the centre of the instrument may coincide with the given angular point in the same line. This done, press the protractor gently down, which will fix it in position by means of very fine points on its under side. It is now ready to lay off the given angle, or any number of angles, that may be required from the given point, which is done by turning the pinion *d* till the opposite vernier reads the required angle. Then press the branches *e e* gently down, and they will cause their points to make the punctures in the paper, at opposite sides of the circle; which being afterwards connected, the line will pass through the given angular point, if the instrument was first correctly set. In this manner, at one setting of the instrument, any proposed number of angles may be laid off from the same point.

PLANNING EXTENSIVE SURVEYS, IMPROVED FORM OF THE FIELD BOOK, &c.

GENERAL DIRECTIONS FOR PLANNING EXTENSIVE SURVEYS.

Provide a sheet of paper, mounted on canvass of the proper size to contain the survey. This may be easily ascertained from the lengths of the longest lines in the survey, and the scale to which it is intended to be laid down. Draw the first line with the proper bearing, which may be determined by the compass of the theodolite, making proper allowance for the variation, as already noticed in the description of surveying instruments. If the survey has been made by the chain only, the bear-

ing may be found by a common pocket compass, or by tying a line to the first or base line in the direction of the sun at 12 o'clock.

If the first or base line be a very long one, a straight-edge should be provided of equal length, for repeatedly splicing the line, as it is termed, with a short straight-edge will almost invariably throw it out of the right direction, and if the first line be longer than any straight-edge that can be procured, (the author has had lines in maps of his surveys nearly 20 feet in length), in this case, stretch a strong silken thread in the proper direction of the line, and carefully puncture the drawing paper in several places, in the exact direction of the thread, the line may then be correctly drawn, with a short straight-edge, from one point or puncture to another.

The first line being laid down, by one or other of these methods, the stations, crossings, &c., marked thereon, take separately, (in the beam-compasses if the extent of survey require them), the second and third lines, or any other two convenient lines that form a triangle with the line already laid down, and, from the proper stations as centres, describe arcs intersecting each other; thus giving the position of the first triangle, which must now be proved by drawing one or more of the secondary lines from their proper stations therein. In the same manner proceed with the other triangles, formed on the first and other lines, till all the lines are laid down.

If the surveys have been made with the help of an angular instrument, as the theodolite, &c., the angles must be laid off at their proper stations, proving the work as it proceeds.

It may here be proper to observe, that in extensive surveys, the lines measured each day must be laid down, and proved at night, that any error that may have occurred may be corrected before the work has proceeded too far; otherwise the correction will involve greater trouble.—The lines of the survey being thus laid down, all the fences, roads, footpaths, rivers, brooks, ponds, bridges, towns, villages, and detached buildings of every kind, as well as every remakable object, must next be laid down in pencil, and the fences and outlines of buildings, roads, rivers, &c., drawn with Indian ink; thus finishing what is called the rough plan.

MR. RODHAM'S IMPROVED FORM OF THE FIELD BOOK.

As an example for practice, a plan with the form of the field book, as given by the late Mr. Rodham, of Richmond, Yorkshire, is presented in the plate facing page 82. This form of the field book was first published by Dr. Hutton, above 50 years ago was then generally adopted, and has not since

been materially improved; the fences and other objects in the survey being sketched therein, on the right and left of the chain line. The author has here departed from the original form by numbering the lines, as being more convenient for reference in large surveys than the method of numbering the stations or marking them with the letters of the alphabet, as recommended by Mr. Rodham.

In the plan referred to, the first line, No. 1, commences at a station in the turnpike road, near N. W. wing wall of the bridge crossing the river Ouse, and is ranged in a north-westerly direction, and so as just to avoid the wood, No. 1; the numbers of this and the other lines being placed in small circles on the lines, to prevent their being mistaken for the numbers of the fields and other enclosures, the same numbers being also placed on the left of the station lines in the field book. Stations are left in line 1 at 250, 1260, 1890, 2335, 2875, 3720, and 4700 links for secondary lines on the right and left of it: this line, having crossed seven fences as shewn in the field book, terminates, in the northern boundary fence of the estate at 4726 links: it will also be seen that the direction of the straight fence on the south of the orchard and homestead, No. 11 is taken, on the left of this line, at 1424 links. The stations referred to, which are afterwards used in the survey, are put down on the right of the station line in the field book, opposite their respective distances; although they might with equal propriety be put immediately above their respective distances in the station line. Some surveyors prefer this latter method, as the stations do not thus interfere with the space allotted to the offsets on the right of the line: the student is, therefore, at liberty to adopt which method he chooses.

Line 2 commences outside the north-western angle of the estate, crosses the boundary at 24, runs through the station at 4700 in line 1, and terminates at the eastern fence of the turnpike road, stations being left therein at 2360 and 3184, and offsets taken therefrom to the northern boundary fence of the estate, which fence is sketched in the field book.

Line 3 begins from ⊙ at 3184 in line 2, turning to the right thereof. This beginning of line 3 is thus marked in the field book:—"from ⊙ at 3184 ┌," the symbol ┌ denoting that line 3 turns to the right of line 2, the symbol ┐ being used when the line turns to the left. This line runs near the turnpike road, for a considerable distance, the two fences of which are taken therefrom by double offsets, as shewn in the field book; and, after crossing the occupation-road between fields 8 and 9, meets the

⊙ at 1260 in line 1, the direction of the straight fence between fields 4 and 8 being taken by a short tie line, as shewn in Prob. IX., Chap. III. The length of this line is 3074 links, and with lines 1 and 2 constitutes a triangle, which may be laid down in the usual way: thus,

Line 4 starts from ⊙ at 250 in line 1 ┌, and closes on ⊙ at 2097 in line 3, taking the turnpike road by double offsets.

Line 5 starts from ⊙ at 1420 in line 3 ┌ and meets ⊙ at 2360 in line 2, taking the western fence of field 3 by offsets.

Line 6 and 7 complete the survey or filling up, as it is usually called, on the north-eastern side of line 1. It is here proper to observe that the four last lines must all fit, or very nearly so, between their proper stations, when laid down on the plan; otherwise some error or errors have been made, either in the measurements, or in the entries in the field book, or in laying down the lines, which must be corrected before the work proceeds any further.

The next main line 8, starts from the beginning of line 2 ┌, of which proper notes are made in the field book, and ends on the north bank of the river Ouse, near the south-west corner of the estate, taking the western boundary thereof by means of offsets throughout its whole length, as shewn by the field book.

Line 9 starts from ⊙ at 4713 in line 8 ┐, and meets ⊙ at 459 beyond ⊙ at 250 in line 1, the distance between these stations being measured on finishing line 9, as it was not foreseen that a ⊙ was required for this line, when line 1 was measured, the entry being made as shewn in the field book.

The remainder of the field book is here put down without sketching the fences, &c., which the student can readily do himself for practice from the plan.

	cross 24	△	post
(10)		1250	
36	37	1179	13 farm house
54 out buildings	37	1160	
188	38	1008	
36	38	972	12 54
	0	740	43
	8	700	62
	cross 56	608	fence
straight fence	47 40	500	
286	48 44	283	
	From	⊙ at	1890 in (1) ┐

	62	2520	
	cross straight fence 89	2032	219
		1552	154
		1380	70
	cross straight fence	1145	88
	buildings { 31		
	38	902	
		1233 ⊙	(10)
		861	
	building { 39	850	cross fence
	36, 40	706	
(13)		530	70
	cross	501	fence
	79	460	
	71	100	
	cross	92	fence
	From	⊙ at	820 in (11) ⌐

	To	⊙ at	900 in (8)
		1077	
	cross	1032	fence
(12)		900	29
		780	57
		550	31
	From	⊙ at	3720 in 1 ¬

	To	⊙ at	2875 in (1)
		1526	
		1436	58
		1040	71
(11)		890	98
		820	⊙
		700	69
		560	36
		200	58
	cross	74	fence
	From	⊙ at	1740 in (8) ¬

	cross	2174	fence
		to ⊙ 2560	
		2045	
(10)		1950	
	98	1316	28 fence

To	⊙ 314	to left of ⊙ at 250 in 1
	1458	
	870	⊙
cross straight fence	786	31
(16)	700	24
	500	18
	300	58
	150	106 R. Ouse 125 links wide.
From	⊙ at	1310 in (15) ⌐

River crosses {	1440	
	1360	
	1310	⊙
	1200	121
(15)	1025	159
	880	161
	750	115
cross straight fence 34	610	41
Wood		
cross straight fence 26	264	28 R. Ouse 125 links wide.
	150	91
From	⊙ at	1660 in (9)

To	⊙ at	4713 in (8)
R. Ouse 125 links wide.	783	
(14) 18	600	
86	440	
124	300	
96	140	
From	⊙ at	2912 in (13) ⌐

River Ouse {	3140	} crosses
	2926	
(13)	2912	⊙
900 ⊙	56 ⊙	on (9)
	2712	

	cross straight	538	fence (wood)
	to	⊙ at	870 in (16)
(17)	R. Ouse 125 links wide.	447	
	36	400	
	22	250	
128	4	68	cross
Bridge	12	52	
120	12	10	
	From	⊙ at	0 in (1)

After a rough plan has been completed, the contents of all the fields, &c., must next be found, by the methods already given in Chaps. II. and III. The scale to which the plan referred to is drawn, is 10 chains to one inch; it being drawn to this scale to accommodate it to the size of a page of this work, but for the purpose of finding the contents with accuracy, the scale to which the rough plan is drawn, should not be less than three or four chains to one inch, otherwise the several parts of the plan cannot be measured with accuracy. The fine plan may next be made to any required scale that may be thought most convenient. Fine plans are of various scales, from 5 to 20 chains to an inch, according to the size of the estate, parish, or manor, or to the desire of the proprietor or proprietors. The fine plan is reduced from the rough one by the pantagraph, or by any of the other methods already given, and is accompanied by a book of reference, containing the names and contents of the several fields and other enclosures, the names of the occupiers, and whatever other particulars, concerning the estate, parish, or manor, that may be required. If the estate be a small one, the reference may be put on one side of the plan. The name of the estate, &c., is usually inscribed with large ornamental letters in a vacant corner of the plan, with the scale to which the plan is drawn.

From what has been already explained with reference to the method of laying down plans, keeping the field book, &c., the student will now have no difficulty in conducting such extensive surveys of parishes, estates, manors, as are referred to at the end of Chap. III., whatever be their varieties of shape; since all surveys made with the chain only, are continued systems of triangulation, or the prolongations of all or some of the sides of the fundamental triangles being made the bases of the further extensions, as may be seen at pages 60, 61, where the

lines for the surveys of the parishes of Woolbedding and Lodsworth are given.

Before quitting this subject, it will be proper to remark, that when two, three, or four surveyors are employed in the survey of a large estate, parish, or manor, it is advisable to divide it into four parts by two large base or main lines crossing one another, at any convenient angle, as near the centre of the work as can be judged by roughly examining it. The main lines must next be tied, as near their extremities as convenient, by four other main lines; which, when laid down, will constitute a proof of the basis of the survey, and will form four large triangles, each surveyor surveying one of these large triangles or spaces cut off by the two first lines. The work, in this case, will be as well connected as if it had been done by one surveyor. The system of fundamental lines in such a survey will nearly resemble that proposed by the Tithe Commissioners for their surveys for first class maps of parishes, for the purpose of the commutation of tithes. This system of lines has been already referred to at p. 59, Chap. III.

CHAPTER V.

EXTENSIVE SURVEYS OF VARIOUS KINDS, EITHER WITH OR WITHOUT THE THEODOLITE.

PREVIOUS to undertaking any extensive projects in engineering, as railways, canals, harbours, the improvement of the navigation of rivers, &c., the district or country through which the railway, canal, &c., is proposed to be formed, must first be surveyed, and an accurate map made thereof, to exhibit the surface required to be occupied by it, and show how far and in what manner the different properties passed over, or near it, may be damaged, if not wholly required by the undertaking, by its cuttings, embankments, &c., if a railway, canal, or harbour; also, by the severance of estates, removal of buildings, diversion of roads, brooks, and watercourses, by the drainage of wells, ponds and watering places, or by filling them up.

When the intended engineering project is laid down on a map of this kind, and, if a railway or canal, the vertical section thereof accompanying it (see Plate III. at the end of the book), the damage done by it may be readily determined, and the diversion of road, watercourses, &c., contrived in the most convenient manner; in order that the several parties affected may be satisfied.

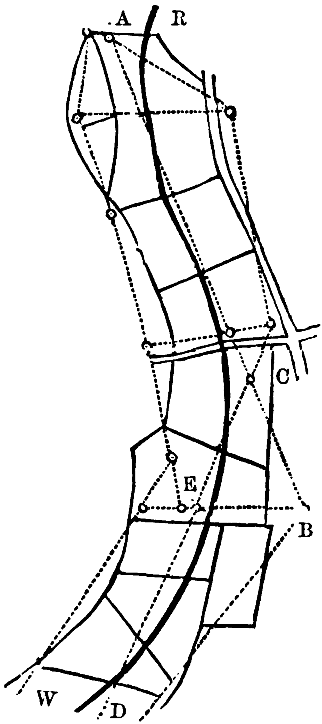
RAILWAY SURVEYS.

The survey given at the end of Chapter III, viz., that of the Parish of Lodsworth, is adapted to railway surveying, on account of its long and narrow form; and by tieing other base lines to AB, in that survey, it may be continued to any extent required. But previous to making surveys of this kind, the line of the proposed railway, as far as it can be determined by trial levels, (see Levelling, Part II., Chap. I.) must be roughly delineated, on such a map of the part of the country or district through which it is proposed to pass, as can be procured; (if the railway be in England or Ireland, Ordnance maps are the best for the purpose) with which the superintendent of the survey must be provided that he may know what part or parts of the country is required to be surveyed for the intended project, and in what direction the main lines ought to be taken, which direction must be as near the proposed line of railway as the obstructions arising from woods, rivers, &c., will admit.

Surveys, containing all the required details for this purpose, may be obtained from the first class parish maps, made under the direction of the Tithe Commissioners. These maps may be generally considered as specimens of accuracy, with the exception of a few that were surreptitiously manufactured by disingenuous surveyors; (see pape 61) but as it was optional to the parish authorities whether they would have first or second class maps, they more commonly choose the latter, on account of the expense being less. The great mass of those second class maps, being partly made or compiled from old and incorrect maps, and partly surveyed by unskilful surveyors, are little better than mere sketches of the parishes of which they are put forth as the maps; so much is this the case, that in making surveys for railways, the author, in running a line of little more than a mile in length, has found them to deviate upwards of ten chains from their true position, while fields shewn by the reference to contain about equal contents, were, as shewn by the map, one more than double another in size. These maps are, therefore, utterly worthless for the work in question, for which nothing less than the most accurate surveys are requisite.

When, therefore, a new survey is required to be made, range the first base line, fixing station flags therein at the most convenient points, also other station flags, to the right and left of the main line, in the direction of fences, roads, rivers, &c., except where there are natural marks that will answer the same purpose as the flags. The measurement of the base line,

and the filling up of the survey on the right and left may then proceed, in the same manner, as already shewn in the narrow part of the Lodsworth survey just referred to. When the first base or main line begins to leave the direction of the proposed line of railway, a second main line must then be set out, from a station 10 or 15 chains short of the extremity of the first main line, that the two main lines may thus be effectually tied to each other; after which the survey may be continued along any number of main lines. Sometimes obstructions prevent the effectual tieing together of the main lines, in this case the theodolite must be used, as shall be shewn hereafter. The width of the survey should be from 5 to 20 or 30 chains; the greater widths being where it has not yet been settled where the line of railway shall pass, or where there are curves therein, or some other engineering difficulty.



The annexed figure represents a survey of this kind; in which the thick, curved line R W, is the projected railway; A B the first base line, which at B begins to leave the direction of the line of the railway. At ⊙ C in the first main line another main line C D is set out, crossing and recrossing the railway. The line C D is connected with AB by the tie line BE. In the same manner the next main line may be connected with C D, and the survey conducted thus to any extent or in any direction. The filling up of the chief parts of the survey is shewn by the dotted lines on both sides of the main lines.

ENGINEERING AND OTHER SURVEYS BY THE THEODOLITE.

The use of the theodolite is either preferable, or absolutely necessary, both in engineering and other surveys; the chief cases of which are the following:—

1. In the surveys of woods, lakes, harbours, &c., by ranging a system of lines round them, and connecting them by taking the angles at their several points of junction.

2. In the surveys of winding roads, rivers, &c., especially where they are obstructed by woodlands, buildings, &c., on both sides, by taking the angles, as in the first case.

3. In the surveys of the streets, of parts or the whole of cities, and large towns.

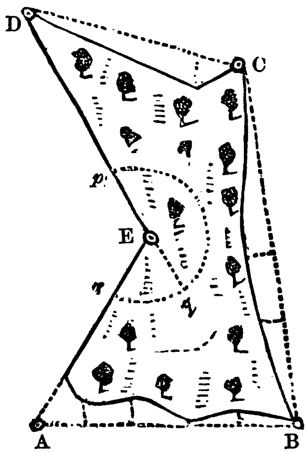
4. In determining the positions and distances of several stations, in an extensive engineering or other survey, by taking angles to them from two or more stations, the distances of which are known.

5. In railway surveys, &c., by connecting the main line with angles, as in the first and second cases, by determining the inaccessible parts that may occur, as in the fourth case; in ranging a long line over high summits. This case frequently embracing all the four preceding ones.

Problem I.

TO SURVEY WOODS, LAKES, HARBOURS, &c., BY THE THEODOLITE.

1. Required the plan and area of the wood represented in the following figure, the field notes, adapted to the theodolite, being given.



Fix station flags round the wood, so that the lines compassing it may be as near it as possible, for the convenience of

taking the offsets; the stations at the same time being made on proper ground for fixing the theodolite. Let A B C D E be the stations, and the field book as below.

	to ⊙ C	
0	2678	
101	1400	
119	800	
0	000	
From ⊙ A	81° 29′	to ⊙ C
	⊙ B ┐	
	to ⊙ B	
0	2302	
99	1800	
0	1100	
202	600	
218	320	
perpr. to corner	225	
Begin at	⊙ A	go E.

From ⊙ E	58° 23′	to ⊙ B
	to ⊙ A	
	1793	
0	1350	
0	000	
From ⊙ D	241° 38′	to ⊙ A
	⊙ E ┌	
	to ⊙ E	
0	1790	
0	000	
From ⊙ C	46° 51′	to ⊙ E
	⊙ D ┐	
	to ⊙ D	
0	1898	
237	200	
0	000	
From ⊙ B	111° 39′	to ⊙ D
	⊙ C ┐	

THE METHOD OF TAKING THE ANGLES, &c.

It will be seen, from the field notes, that the line AB is first measured, as a base for the plan. The first angle ABC is then taken, which is found to be 81° 29′, and shews the direction of the second line B C. In taking this angle, the theodolite is fixed directly over ⊙ B; the two zeros of the horizontal plates being then clamped together, the object glass of the telescope is directed to the flag or other mark at ⊙ A, and the whole instrument clamped; the upper plate is now unclamped, and the telescope directed to the flag at ⊙ C, when the angle ABC is found to be 81° 29′, as shewn in the field notes. In a similar manner, the angles at stations C and D are respectively found to be 111° 39′ and 46° 51′, the three lines B C, C D, D E each bending respectively to the left of the line preceding it. The angle at ⊙ E is found to be 241° 38′, which being greater than 180°, that is, greater than the semicircle *p q*, shews that the line E A turns to the right. Finally, the angle at ⊙ A is found to be 58° 23′, shewing that the line A B turns to the left of E A. Thus the magnitude of the angle shews whether the

new line inclines to the right or the left of the old one, *the new line turning to the left of the old one, when the angle is less than* 180°, *and to the right when greater, the zero of the instrument being always directed to the commencement of the old line:* therefore, the bearing of all the lines, except the first, may be omitted in the field notes; the remainder of which, being similar to those already given, need no further explanation.

PLANNING AND PROVING THE WORK.

Draw the base line A B in the given direction, indefinitely, and lay off the given length 2302 links thereon. Place the centre of the protractor at ⊙ B, with its straight side close against A B, and prick off 81° 29′ from its end towards A; then, through ⊙ B and the protractor-mark, draw BC, making it the given length 2678. Lay down similarly the two following lines with their angles C and D. The angle at ⊙ E is 241° 38′, therefore E A must turn to the right, and the angle to be laid off at E is 360° − 241° 38′ = 118° 22′; or, if a circular protractor be used, the whole angle may be laid off at once, and E A being drawn, must reach to ⊙ A, where the survey began, or so very near to it that the error may be considered immaterial; but if it do not reach to ⊙ A, by a considerable distance, there has been an error either in taking the angles or measuring the lines. But since *the sum of all the interior angles of a polygon is equal to twice as many right angles as the figure has sides, lessened by four right angles*, and since the given figure has five sides, the sum of all its five interior angles will be $= 5 \times 2 - 4 = 6$ right angles $= 90° \times 6 = 540°$. This will be found to result by adding all the angles of the figure, as below.

Angle at B =	81°	29′
——— C =	111	39
——— D =	46	51
——— E =	241	38
——— A =	58	23
Proof, as respects the angles	540°	0′

The above result shews that the angles have been accurately taken; if, therefore, the work do not close, that is, if E A does not reach to ⊙ A, the error is in the measuring of some of the lines, or in making a wrong entry in the field notes, which may now be readily detected, if required.

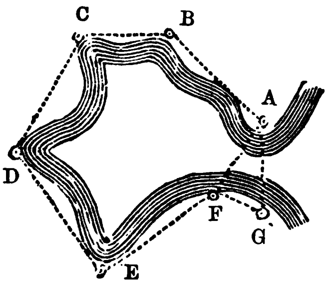
When the work is planned the content will be found to be 37*a.* 3*r.* 4*p.*

It will be seen that the above survey might have been effected by the method given in Prob. VIII., Chap. III.; but it is here assumed that the lines compassing the wood cannot be prolonged for the purpose of measuring tie lines, on account of obstructions, as is often the case.

Lakes, Meres, and large Ponds are surveyed and planned in the same manner as the wood just given.

2. The following figure represents a gulf or inlet of the sea, the survey of which is required to adapt it for the purposes of a harbour for ships.

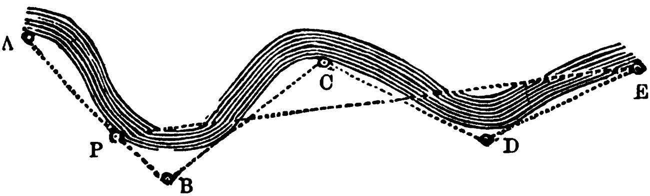
The coast, here shewn, is the boundary of high water; the survey begins at ⊙ A, station-flags being fixed at B, C, D, E, F, and G, and angles taken at A to F and G. The line B A, being first prolonged backwards to high water mark is then measured to B, and the angle A B C taken. Similarly all the succeeding lines are measured, and the angles taken; also at F and G the angles are taken to ⊙ A, and the line F G prolonged to high water mark, all the offsets being taken as the work proceeds. The figure may now be laid down, precisely as in the last example, the magnitude of the angles shewing the directions of the lines, and the lines A F, A G, which could not be measured on account of the great width of the entrance of the harbour, proving the work by means of the angles, taken at A, and F, and G.



Problem II.

THE SURVEYS OF ROADS AND RIVERS.

1. The following figure represents a river, the survey and plan of which is required, for the purpose of improving its navigation, making locks, &c.



Station flags being set up, at or near the principal windings of the river, as at A, B, C, D, E, the line A B is measured, and the offsets taken to the nearest shore of the river, its widths, if very great, being determined by Prob. IV., Chap. III.; or by throwing a leaden ball across the river, with a slender cord attached thereto at one end, and holding the other end in the hand, and then drawing it back, and measuring the length of cord, required to reach the opposite shore. This last method is impracticable where the width of the river is very great. A flag is left at ⊙ P in A B, where the sight, in the general direction of the river, is unobstructed for a considerable distance. The measuring of AB being now finished, the angle at B is taken, which, being less than 180°, shews that B C turns to the left. On measuring to C the angle there is found to be greater than 180°, shewing that CD turns to the right; and thus the work proceeds to ⊙ E, where an angle is taken to the now-distant flag at P. This last angle will prove the accuracy of the work when laid down.

If the width of the river be very great and unequal, a similar system of lines must be used on the opposite shore; otherwise, a correct map cannot be obtained, the two systems of lines being connected by finding occasional widths of the river, as already stated; thus the line B C may be prolonged across the river to connect another system of lines, if required.

2. If a *road* be represented by the winding figure, in the last example, it may be surveyed precisely in the same manner, excepting that it would be more convenient to have the system of lines A B C D E upon the road, instead of the side of it, that the offsets may be readily taken to the right and left of the several lines, recollecting to leave a flag, or some other prominent object, in or near the first line, as at ⊙ P, in the last figure, to which an angle may be taken, after the survey has proceeded a considerable distance, to prove the work.

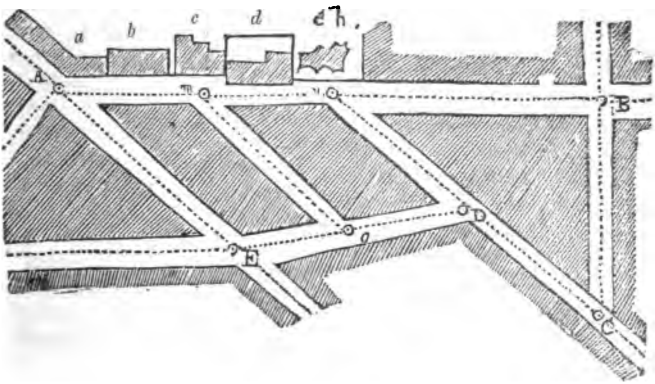
The method of planning either a river or road will be sufficiently clear, from the first example in Prob. I.

The map of a river or road being thus obtained, if the area be required it may be readily found by the methods previously given, or, by taking widths at the end of every chain, estimated along the middle of the river or road, and taking a mean of the widths, which, multiplied by the length, will give the area sufficiently correct for most cases.

PROBLEM III.

THE SURVEY OF A PART OR THE WHOLE OF A CITY OR LARGE TOWN.

CASE I. Commence the survey at the meeting of three or more of the principal streets, through which the longest prospects can be obtained, for the purpose of laying out the main lines. Having then selected a proper station, fix the theodolite thereon, making a line in one of the principal streets a base line, and directing it to some prominent or well-defined point as to a projecting corner of a house, or to a lamp, or railing-post, or to the right or left side of a conspicuous door or window, which point must be described in the note book, that it may be remembered. Angles must first be taken between the base line and the other lines, diverging from the first station, defining carefully the directions of these lines, as in the case of the base line. This done, measure these lines with the chain, taking offsets to all corners of streets, bendings, or windings, and to all remarkable objects, as churches, halls, colleges, eminent buildings, &c.; also, defining the extent of buildings belonging to each separate owner, or joint-owners, especially if such buildings are required to be taken down for engineering purposes, or for improvements: at the same time recollecting to



leave stations opposite the ends of the streets to the right and left, and to take the angles of the directions. This operation must be repeated on the other main lines, till the survey is completed.

Thus, having fixed the theodolite at A, take the angles of lines meeting there, referring them to the base line A B, that the magnitude of the angle may shew their direction: then measure A B, taking offsets to the buildings of different proprietors, as to the buildings marked *a*, *b*, *c*, &c., on which the dimensions of their several parts, yards, &c., must be put in the note book, that they may be accurately mapped, preparatory to their valuation, if required to be taken down for engineering purposes, or for parish rating; stations being left at *m* and *n*, for the lines in the streets on the right, and the angles of their directions taken; strong iron pins, driven into the crevices of the pavement, being the usual station marks in large towns. The measurement thus proceeds to ⊙ B, where the angles of the streets diverging from it are now taken. The line B C is next measured in like manner, and the angles taken at ⊙ C; after which the measurement proceeds to ⊙ *n*, in the base A B; thus constituting the triangle *n* B C, a station being left at D in C *n*. From ⊙ D the survey proceeds to ⊙ E, and from thence to ⊙ A, where the work commenced, a station having been left in the line D E at *o* for the line to ⊙ *m*. In this manner the survey may be continued to any required extent.

This survey, so far as it has been here shewn may be plotted independent of the angles taken with the theodolite, by first laying down the triangle *n* B C, and then determining the position of ⊙ E by intersection from stations A and D, when the line *m o* will prove the work. But it rarely happens, in the practice of town surveying, that a fundamental triangle can be obtained, sufficiently large to lay down the work in this manner; it is merely here shewn that such a case is possible; for had the street, in which the line C *n* is measured, been so bent as not to admit a right line along it, the use of the theodolite would have been indispensable in this survey. Assuming, therefore, that C *n* is not a right line, but bent at ⊙ D; then, in the five-sided figure A B C D E, the accuracy of the measurement of the angles may be proved, by taking the sum of all the interior angles as in Ex. 1, Prob. I.; and the work further proved, by the closing of the lines at ⊙ A, as well as at the several other stations.

The form of the field (or rather town) notes in this case, is the same as those already given in the field book, Plate p. 82, excepting that the entries must be made at a sufficient distance apart, that sketches of buildings, yards, gardens, &c., may be clearly made, with the measures of their several parts put on

them in links, and the angles must be noted in the same manner as in the example of the wood, Prob. I.

Case II. If a very large town be required to be surveyed, the best method is to measure a base line of considerable length, on elevated and open ground, on the outside of the town and at two stations at its extremities take horizontal angles to the towers of churches and other lofty buildings in the town, and the intersections of the lines of sight, from these angles will determine the positions of the towers, &c.; which may then be made stations for the survey of the several streets, which may now be conducted in the manner shown in Case I. Moreover a third station must be taken in the line thus measured, at which angles must be taken to all the towers, &c., which angles, being laid down, their lines of sight will pass through the intersections of the lines of sight taken from the other two stations, if the work be correct, otherwise an error has been made in taking some of the angles, which must be corrected, before the survey of the streets, &c., be commenced.

The several distances of the towers and other lofty buildings may be calculated by Trigonometry (see *Trigonometry, Weale's Series*), and the several lines, or triangulation, connecting the said towers, &c., may thence be laid down, without plotting the exterior base line and the lines of sight, taken from it. This last method of proceeding is shown in the following Problem, and is adopted by the Ordnance authorities.

Case III. If the town be long and narrow, with straight openings across, either through straight streets, or partly through streets and gardens, a triangulation may be formed on the open ground outside the town and the main lines may be connected by other lines passing through these openings, in which stations may be obtained for the survey of the other streets. Such a survey would resemble the parish-surveys, already described.

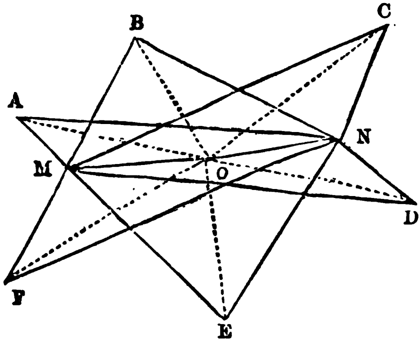
This method was partly adopted by the author in the survey of Dover for Rowland Rees, Esq., Architect and Surveyor for that borough, the theodolite being only used in those parts where this method was impracticable.

Problem IV.

TO DETERMINE THE POSITIONS OF SEVERAL DISTANT POINTS, OBJECTS, OR STATIONS, BY TAKING ANGLES AT TWO STATIONS AT THE ENDS OF A GIVEN LINE.

Let A, B, C, D, E, F be six stations, the positions of which

are to be found. Measure a line M N on level ground, and such that all the stations may be seen from M and N, and at each of the stations M and N, take angles with the theodolite to all the stations; its zero, in taking the angles at each station, being directed to the opposite end of the given line, that the magnitude of the angle may determine the direction of each line of sight to the distant stations. The line M N being then laid down, and the angles taken at its extremities, the intersections of the lines of sight M A, M B, &c., and of NA, NB, &c., will determine the positions of the several distant points A, B, &c., from whence their distances may be found, and made base lines for the further extension of the survey, if required.

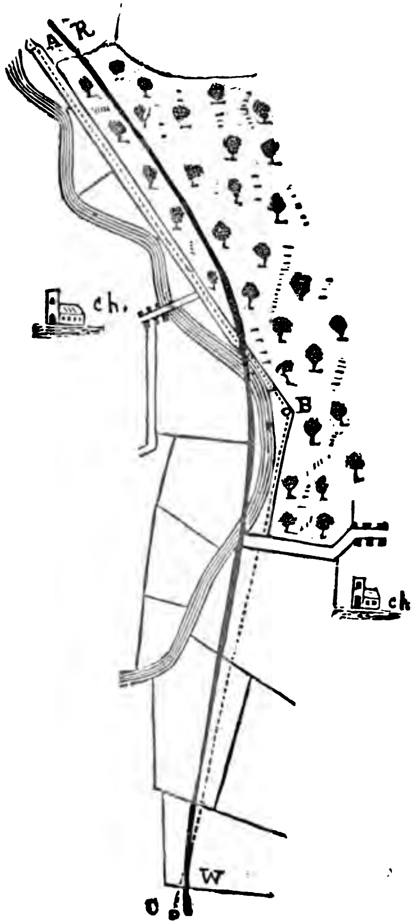


The accuracy of the work may be proved by taking a ⊙ *o*, at any point in M N, and taking from thence angles to all the distant stations; which angles, being laid down, their lines of sight will pass through the intersections A, B, &c., if the work has been correctly done.

When all the distant stations or objects cannot be seen from two given stations, then three stations may be taken, or as many as are necessary; connecting the stations thus used, by measuring their distances, and proving their positions by other lines, or by angles: the zero of the theodolite in taking the angles to the distant stations being always directed to the last given station. Moreover, the angles to all remarkable objects that can be seen from two given stations may be taken at the same time; thus may their positions also be determined by the intersection of two or more lines of sight. In this manner very extensive surveys may be effected, occasionally checking the accuracy of the work by measuring the distances of some of the distant stations, where it can be the most conveniently done. The distances from A to B, from B to C, &c., may be calculated by Trigonometry, as in the preceding Problem.

TO SURVEY A DISTRICT WHERE OBSTRUCTIONS OCCUR, FOR A RAILWAY, CANAL, ETC.

In the figure, RW is a portion of a projected railway, and AB, B C the main lines to survey that portion of the district occupied by or affected by the railway. The first main line AB runs along a road, till it is obstructed by a large wood on the right, a brook being close to its left, and, since the direction of the railway changes near the point of obstruction to the left, a new main line B C is taken. Here the use of the theodolite is indispensable to take the angle of the two main lines AB, BC at B, as it is impossible, on account of the obstructions in question, to obtain tie lines to connect the main lines, excepting with great trouble. The parts of the survey adjoining the wood and the brook may be filled up in the usual way, while angles must be taken from two or more stations, to determine the posi-



tions of churches, and other remarkable objects on the right and left of the railway, by intersection; it being unnecessary, at the same time, to measure their distances, especially if those distances from the railway be considerable, their positions being only required to shew more clearly the locality of the proposed railway to those interested therein. Also, at the same time, the angles of fences, or other parts of the actual survey that may be in the same direction of the churches, &c., may be conveniently taken, which will thus save the trouble of measuring tie lines. As the railway again leaves the direction of the main line B C at C, a third main line will be required (this line is not shewn in the figure); which may be either tied to B C, or its angle of inclination thereto be taken with the theodolite, according to convenience. The filling up of the part of the survey to the right and left of the part of B C, towards C may be readily effected by the help of the prismatic compass, or box sextant, on account of the fences being straight, and the fields large; the angles which these straight fences make with B C, being taken, and the lengths of the fences measured to the meetings of other straight fences.

CHAPTER VI.

SECTION I.

THE VARIOUS METHODS OF LAYING OUT ANY GIVEN QUANTITY OF LAND IN ANY PROPOSED REGULAR OR IRREGULAR FORM; OF SEPARATING ANY REQUIRED QUANTITY OF LAND FROM AN ENCLOSURE OR COMMON; AND OF DIVIDING LAND OF VARIABLE VALUE AMONG SEVERAL CLAIMANTS, IN PROPORTION TO THEIR RESPECTIVE CLAIMS.

PRELIMINARY PROBLEMS.

PROBLEM I.

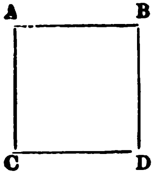
To lay out a given quantity of land in the form of a square.

RULE.—Reduce the given quantity to square links, by the table of square measure page 15; and the square root of the result is a side of the required square, in links.

EXAMPLES.

1. Lay out $2a.\ 1r.\ 5\frac{1}{2}p.$ in the form of a square.

$$\begin{array}{ll} 2a. & = 200{,}000 \text{ square links.} \\ 1r. & = 25{,}000 \\ 5\frac{1}{2}p. & = 3{,}437 \end{array}$$



$$\begin{array}{r} \overline{228{,}437}(478 \\ 16 \\ \hline 87)684 \\ 609 \\ \hline 948)7537 \\ 7584 \\ \hline \end{array}$$

links nearly, = a side of the square A B D C.

2. Required the side of a square, that shall contain $3a.\ 3r.\ 28p.$ *Ans.* $626\frac{1}{2}$ links.

NOTE. A given quantity of land is sometimes required to be laid out in the form of a square for pleasure grounds, &c.

PROBLEM II.

To lay out any proposed quantity of land in the form of a rectangle, the length of which is given.

RULE.—Divide the square links in the quantity of land by the given length, and the quotient will be the breadth of the required rectangle.—If the dimensions are required in feet, the given area must be reduced to square feet.

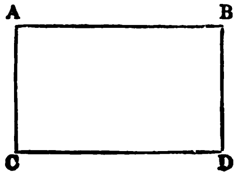
EXAMPLES.

1. Lay out $4a.\ 0r.\ 12p.$ in a rectangular form, the length of which shall be 812 links.

$4a.\ 0r.\ 12p. = 407{,}500$ square links, and

$$\begin{array}{r} 812)407500(501{\cdot}8 \\ 4060 \\ \hline 1500 \\ 812 \\ \hline 6880 \\ 6496 \\ \hline 384 \end{array}$$

links = A C, the required breadth of the rectangle ABDC.



2. The breadth of a rectangular piece of ground is 140 feet; required its length, when its area is 2 acres.

$$\begin{array}{l} 1 \text{ acre} = 43560 \text{ square feet} \\ \qquad\qquad\qquad\quad 2 \\ \hline \end{array}$$

$$14{\cdot}0\left\{\begin{array}{r|l} 2 & 8712{\cdot}0 \text{ square feet in 2 acres.} \\ \cline{2-2} 7 & 4356 \\ \cline{2-2} \end{array}\right.$$

$622\frac{2}{7}$ feet, the length required.

3. The length of a rectangular pleasure ground is proposed to be 1000 feet, required its breadth so that it contain $12\frac{1}{2}$ acres. *Ans.* $544\frac{1}{2}$ feet.

NOTE. Besides the extensive use of this Problem in the division of commons, &c., it is also used to determine the length of frontage of a given quantity of building ground, when its depth or breadth is given; or, to determine the depth, when the length of frontage is given.

PROBLEM III.

To lay out a given quantity of land in the form of a rectangle, the length of which shall have a given proportion to its breadth: that is, the length shall be to the breadth as 3 *to* 1 *or as* 4 *to* 3, *or as* 5 *to* 2, *&c.*

RULE.*—Divide the given area by the product of the two terms of the proportion, and the square root of the quotient, multiplied separately by the terms of the proportion, will give the required length and breadth.

EXAMPLES.

1. Lay out 3 acres of land in the form of a rectangle, the length of which shall be to its breadth as 3 to 2.

$3 \times 2 = 6)300000$ square links in 3 acres

$$\begin{array}{rl} & 50000(223{\cdot}6 \\ & 4 \\ \hline 42) & 100 \\ & 84 \\ \hline 443) & 1600 \\ & 1329 \\ \hline 4466) & 27100 \\ & 26796 \\ \hline & 304 \end{array}$$

links.

$223{\cdot}6 \times 3 = 670{\cdot}8$ length.

$223{\cdot}6 \times 2 = 447{\cdot}2$ breadth.

* Let mx and nx be the length and breadth of a rectangle, which are in the ratio of $m : n$, and a its area; then $mx \times nx = mnx^2 = a$, or $x^2 = \frac{a}{mn}$, or $x = \sqrt{\frac{a}{mn}}$. Q. E. D.

2. Lay out $6\frac{1}{4}$ acres in a rectangular form, the ratio of whose length and breadth shall be as 5 : 2.

$5 \times 2 = 10)625000$ links.

$250 \times 5 = 1250 =$ length.

$\sqrt{62500} = 250.$ $250 \times 2 = 500 =$ breadth.

3. It is required to lay out a rectangular fish pond, the length of which shall be four times its breadth, and its area 10 acres.

Ans. Length 2000 and breadth 500 links.

NOTE. Proportions of this kind are frequently required in laying out ornamental grounds, ponds, &c., in rectangular forms.

PROBLEM IV.

To lay out a triangle, of a given area and a given base, one of the sides of which shall have a given position.

RULE 1.—Divide twice the area by the base for the perpendicular, which erect at any point in the base, and at the extremity of the perpendicular and at right angles to it, range a line till it meet the side given in position, which point of meeting is the vertex of the triangle required.

EXAMPLES.

1. It is required to lay out a triangle A B C to contain $2a.\ 2r.\ 16p.$ on a base A B of 1200 links, the position of A H, of which the side A C is a part, being given.

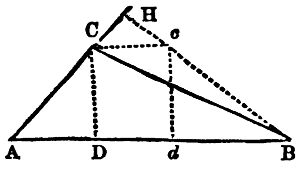
By Rule 1.

$2a.\ 2r.\ 16p. = 260000$ links

2

$1200)520000$

$433\frac{1}{3}$ links$=$CD



On any point d of the given base AB $= 1200$ links, erect the perpendicular $dc = 433\frac{1}{3}$ links; range cC perpendicular to dc till it meet A H in C; then, B C being joined, A B C is the triangle required. For let CD be drawn perpendicular to AB, then, by the nature of the construction, C D $= cd$. Q. E. D.

RULE 2.—Measure HB perpendicular to AH, the line given in position, to the end B of the given base, and divide twice the given area by H B, and the quotient is A C, which, being measured off, will give the point C.

2. Solve the last example by Rule 2, the perpendicular B H being 860 links.

$$\frac{260000 \times 2}{860} = 604{\cdot}65 \text{ links} = \text{A C.}$$

3. Required the perpendicular of a triangle, containing 26*a.* 2*r.* 28*p.*, its given base being 3112 links.

Ans. $1714\frac{1}{8}$ links, nearly.

Note. This Problem is extensively used in the division of lands, commons, &c., as shall be hereafter shewn.

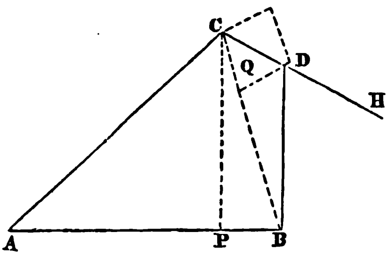
Problem V.

To lay out a trapezium of given area, the positions and lengths of two of its adjoining sides being given, and also the position of its third side.

Rule.—Join the ends of the two adjacent sides, thus forming a triangle, the area of which must be found, which area must be subtracted from the given area, and the remaining area must be laid out in the form of a triangle, on the line joining the two given sides, as base, by Prob. IV.

EXAMPLE.

Let the given area of the trapezium A B D C be 6 acres, A B, A C the sides that are given in length and position, and C H the position of the third side. Join BC; measure the perpendicular CP, which is found = 840 links, and C B = 880 links, A B being = 1200 links; whence the area of the triangle A B C $= 600 \times 840 = 504000$ square links; and $600000 - 504000 = 96000$ = area of the triangle B C D; therefore $96000 \times 2 \div 880 = 218\frac{2}{11}$ links = perpendicular DQ; which, it will be seen, is applied as in Prob. IV.; thus constituting the required trapezium ABDC. CD may also be found as by Rule 2, Prob. IV.



Note. This Problem is also much used in the division of lands, commons, &c.

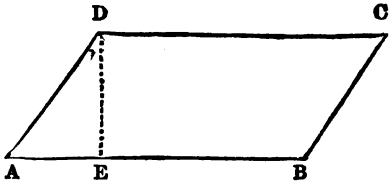
Problem VI.

To lay out a rhomboid of given area, the lengths of its sides being given.

Rule.—Divide the area by the longer side, subtract the square of the quotient from the square of the shorter side, and the square root of the remainder is the distance of the perpendicular from the end of the larger side, which perpendicular, being made equal to the quotient first found, gives the breadth of the rhomboid.

EXAMPLES.

1. The area of a rhomboid is required to be 6 acres, and its longer and shorter sides to be respectively 1200 and 625 links, required the place and length of its perpendicular.



$$12\,00)\underline{600000}$$
$$500 = \text{perpr.} = DE$$
$$625^2 = 390625$$
$$500^2 = \underline{250000}$$
$$\sqrt{140625} = 375 \text{ links} = AE.$$

2. Lay out the figure when its sides are each 4 chains, that is, when it is a rhombus, its given area being 1*a.* 1*r.* 24*p.*

Ans. 350 links = perpendicular, and its distance 193·6 links.

Problem VII.

To lay out a proposed quantity of land in a regular polygon.

Rule.—Divide the area of the proposed polygon, by its corresponding area in the following table, and the square root of the result will be the length of its sides. Multiply the side just found by the corresponding radius, in the column marked radii, and the result will be the radius of the circle that circumscribes the required polygon.

A table of regular polygons, with their names, areas, and radii of their circumscribing circles, the sides of the polygons being unity.

No. sides.	Names.	Areas.	Radii.
3	Triangle	0·433	0·577
4	Square	1·	0·707
5	Pentagon	1·72	0·851
6	Hexagon	2·598	1·
7	Heptagon.........	3·634	1·152
8	Octagon	4·828	1·306
9	Nonagon	6·182	1·462
10	Decagon	7·694	1·618
11	Undecagon	9·365	1·775
12	Duodecagon......	11·196	1·932

Note. If the square of the side of a polygon be multiplied by its corresponding area in the preceding table, the product will be the area of the polygon.

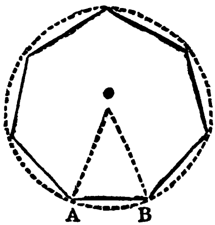
EXAMPLES.

1. Lay out an acre of land in a regular heptagon.

$$\text{Here } \sqrt{\frac{100000}{3\cdot634}} = 165\cdot88 \text{ links} = AB.$$

$$\text{Also } 165\cdot88 \times 1\cdot152 = 191\cdot09 \text{ links} = AO.$$

Construction of the heptagon on the ground.—Provide a strong cord, equal in length to the radius of the circumscribing circle of the heptagon, which in this case is 191·09 links, or very nearly $191\frac{1}{10}$ links; fix one of its ends at *o*, as a centre, and tie the other end to a pointed arrow; stretch the cord to A, and with the radius O A describe the dotted circle, keeping the arrow vertical, and making a visible circular mark with it. Then take the distance A B = 165·88 links on the cord, which being applied seven times within the circle, will just reach round the circumference, thus forming the heptagon required.



2. Lay out 4 acres of land in a hexagon.

Ans. 392·38 links = radius of circumscribing circle and side of the hexagon, for both are equal in this figure.

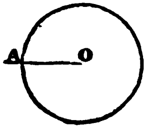
PROBLEM VIII.

To lay out a given quantity of land in a circular form.

RULE.—Divide the given area by ·7854, and half the square root of the quotient will be the radius of the circle.

EXAMPLES.

1. Lay out 4 acres of land so that its boundary may be a circle.



$$\text{Here } \frac{1}{2}\sqrt{\frac{400000}{\cdot7854}} = 356\cdot8 \text{ links} = AO.$$

With the radius A O, the circle may be described on the ground, as in the last Problem.

2. Required the radius of a circle containing half an acre of land. *Ans.* $126\frac{1}{8}$ links.

NOTE. The rules given in the two preceding Problems are the inverse of the rules for finding the areas of the same figures in works on mensuration; and their investigations are given in various works on analytical trigonometry.

PROBLEM IX.

To lay out a given quantity of land in the form of an ellipse.

RULE I.—*When one of the diameters is given.* Divide the given area successively by ·7854 and the given diameter, and the quotient will be the other diameter.

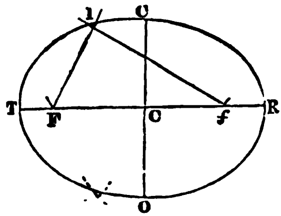
RULE II.—*When the two diameters are required to be in a given proportion or ratio.* Divide the given area successively by ·7854 and the product of the terms of the ratio, and the square root of the quotient, multiplied separately by the terms of the ratio will give the two diameters.

EXAMPLES.

1. Lay out 4 acres in an elliptical form, with a transverse diameter T R of 9 chains.

By Rule I. $\frac{400000}{\cdot 7854 \times 900} = 566 \begin{cases} \text{links} = \text{C O, the} \\ \text{conjugate diameter.} \end{cases}$

Construction of an ellipse.—Lay off from the centre C of the ellipse, on the transverse T R, the distances CF, C*f*, each equal to the square root of the difference of the squares of the two semi-diameters; take a strong flexible cord, equal to T R, and fix its ends at F, *f*; extend the cord to I, so that it may take the position F I *f*, and keeping the cord continually stretched, with an arrow trace the elliptical curve T I C R O, which will be the required boundary.



In this case $CF = Cf = \sqrt{450^2 - 283^2} = 350$ links.

2. Lay out an ellipse to contain 2*a.* 0*r.* 32*p.*, the proportion of the diameters of which shall be as 7 to 4.

By Rule II. $\sqrt{\frac{220000}{4 \times 7 \times \cdot 7854}} = 100$ links; and $100 \times 7 = 700$ links, the transverse diameter, and $100 \times 4 = 400$ links, the conjugate.

NOTE 1. The investigation of the rules used in this Problem, are given in various works on Conic Sections.

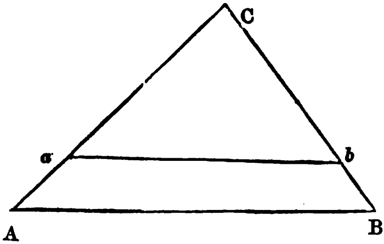
NOTE 2. The three last Problems are chiefly of use in laying out ornamental grounds, as the inner area of squares in cities and towns, shrubberies, plantations in parks, &c.

PROBLEM X.

To divide any proposed quantity of land from a triangle, by a line parallel to one of its sides.

RULE.—Since the areas of triangles are as the squares of their like or homologous sides, we shall have

$$\text{Area } \triangle\, ABC : \text{area } \triangle\, abc :: AC^2 : aC^2.$$



See Theor. VIII., Chap. I.

EXAMPLES.

1. Let the base A B = 2600 links, A C = 2000, and B C = 1600, cut off 6*a*. 1*r*. 24*p*. by the line *ab* parallel to A B.

From the three sides the area of the triangle A B C is found = 1599218 square links,
and 6*a*. 1*r*. 24*p*. = 640000 square links,
the difference = 959218 = area of $\triangle\, abC$.

Then per Rule, $1599218 : 959218 :: 2000^2 : 2399217$; and $\sqrt{2399217} = 1548{\cdot}94$ links $= aC$; whence $2000 - 1548{\cdot}94 = 451{\cdot}06$ links $= Aa$. Again by similar triangles $2000 : 1600 :: 451{\cdot}06 : 360{\cdot}84$ links $= Bb$. Therefore, measure on the ground the distances A *a*, B *b*, just found, respectively, from the points A and B; and range the line *a b*, which will divide the given quantity A B *b a* from the triangular field A B C, as required.

NOTE. In this manner a triangle may be divided into any number of equal or unequal parts, by lines parallel to any one of its sides, by successively subtracting the sums of 2, 3, 4, &c., areas from the area of the triangle, and making the remainders successively the second terms of the proportion.

2. The sides of a triangular field are 900, 750, and 600 links; it is required to cut 0*a*. 3*r*. 28*p*. therefrom, by a straight fence parallel to its least side.

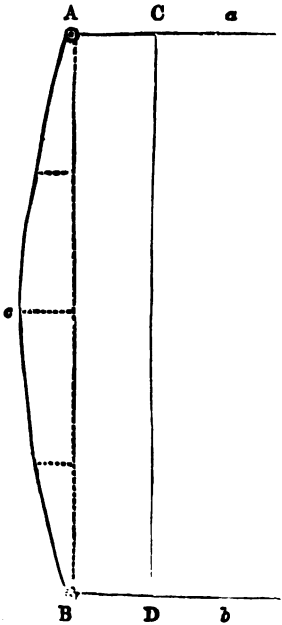
Ans. The distance from the ends of the least side, on the largest and intermediate sides, are respectively 211½ and 176 links.

PROBLEM XI.

To lay off any given quantity of land from an irregular field or common.

RULE.—First find the area of the crooked or irregular part by means of offsets, which area must be deducted from the given area, after which the remaining area must be laid out by means of Problems II., IV., and V.

Case I.—When one side of the field is crooked and two straight and parallel.



EXAMPLE.

Let a A c B b be a portion of a field or common, the side A c B being crooked and A a, B b parallel, the length of A B is 2400 links, and the area of the offset piece A c B is found to be 227500 square links; it is required to cut off 12 acres from the field by a line parallel to A B.

12 acres = 1200000 sq. links
offset piece = 227500 ditto

$$24{\cdot}00\left\{\begin{array}{l} 2)9725{\cdot}00 \\ \hline 12)4862{\cdot}5 \\ \hline \end{array}\right.$$

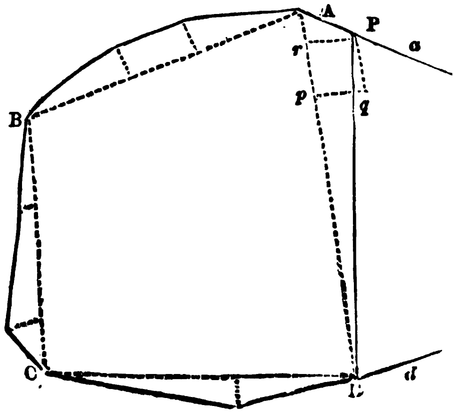
$405{\cdot}2 = AC = BD$

Therefore, measure the distances A C = B D = $405\frac{1}{5}$ links on the ground, and range the straight line C D, then shall the space A c B D C contain 12 acres.

Case II.—When all the sides of the field or common are crooked.

EXAMPLE.

Let a A B C D d be a portion of a field or common, the boundary of which is crooked, it is required to part therefrom 6 acres, by a straight fence running from D towards A.



Measure the four lines AB, BC, CD, DA, taking the offsets on the three first, and finding the

area of the trapezium A B C D in the usual way, which united areas are found to be 570000 square links, the line A D being 810 links.

$$\begin{array}{rl} \text{Hence 6 acres} = & 600000 \text{ square links} \\ \text{Area A B C D and offsets} = & 570000 \text{ ditto} \\ \hline & \end{array}$$

$$81{\cdot}0\left\{\begin{array}{l} 9)3000{\cdot}0 \\ \hline 9)333{\cdot}3 \end{array}\right.$$

$$\begin{array}{r} \hline 37{\cdot}037 \\ 2 \\ \hline 74{\cdot}074 \text{ links} = \text{perpend. P}r. \end{array}$$

Therefore, apply the perpendicular P r = 74·074 links as in Prob. IV., and range the straight line A P; then shall the 6 acres be parted from the field or common as required.

Problem XII.

To divide from a field or common, bounded by straight fences, any given quantity of land, by a line parallel to one of the straight fences.

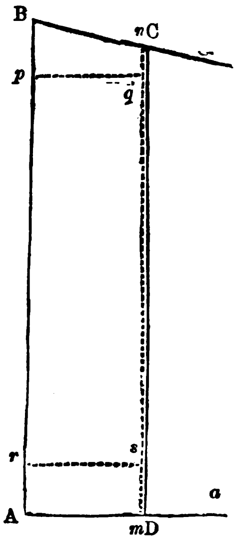
Rule.—Measure the length of the straight fence or line to which the division line is required to be parallel; divide the given quantity by that length, and set the quotient perpendicular to the given straight fence, at two points near the ends thereof, ranging a line through the extremities of the two perpendiculars, and measuring the length thereof; find the area of the trapezoid thus obtained, and take the difference between this area and the given area, which difference, being divided by the last measured line, will give the breadth to be set out perpendicularly from the last measured line, either inwardly or outwardly, accordingly as the difference is in excess or defect of the given quantity.

EXAMPLE

Let a A B b be a portion of a field or common with straight fences, it is required to lay out 10*a.* 2*r.* 16*p.* by a line parallel to A B, the length of which is 2200 links.

$$2200\left\{\begin{array}{l} 2)1060000 = 10a.\ 2r.\ 16p. \\ \hline 11)5300 \end{array}\right.$$

$$\begin{array}{r} \hline 482 \text{ nearly} = p\,q = r\,a. \end{array}$$



Now the perpendiculars $p\,q$, $r\,s$ being set off, and the line $m\,n$ measured, is found to be 2025 links; whence

$(2200+2025)\times\frac{482}{2}=1018225$ sq. links, and

$(1060000-1018225)\div 2025=20{\cdot}6$ links, the distance to be set perpendicularly, at both ends of $m\,n$, to the right thereof, because the area cut off by $m\,n$ is less than the given quanity, and the straight line C D ranged through the extremities of these perpendiculars will be parallel to A B, and will cut off the required quantity.

NOTE 1. The three last Problems are much used in the inclosure of commons, and other waste lands, and more especially Problem XII. as the laying out inclosures with parallel fences contributes very much to the economy of cultivation, by thus allowing the ridges to run from the ends of each inclosure, parallel to the two parallel fences.

NOTE 2. Those who are accustomed to operations in analytical trigonometry, will find the following formulæ much more expeditious for laying out land by means of parallel fences, as in Prob. XII., especially where a great number of successive inclosures are required to be laid out:—Let $AB=a$, $\sigma=$ sine $\angle A$ $s=$ sine $\angle B$, $S=$ sine $(\sigma+s)$, and $A=$ area ABCD, which is required to be cut off; then,

$$BC=\frac{1}{S}\left(a\sigma-\sqrt{\frac{\sigma}{s}\,(a^2s\sigma-2AS)}\right),$$

$$\text{and } AD=\frac{s}{\sigma}BC.$$

Whence both B C and A D become known without any preliminary measurement, except that of the side A B and the angles A and B: and by adding the next area to be cut off to A, and calling the sum B, and then substituting B for A in the formulæ for B C, the next following distances B b and A a may be found; and so on for any number of inclosures.—When A and B are supplemental angles, the lines A a, B b, will be parallel, which makes the Problem extremely simple; and when the sum of the angles at A and B are greater than two right angles, S becomes negative, and the lines A a, B b, will meet on the other side of A B. In this case the formulæ will become

$$BC=\frac{1}{S}\left(\sqrt{\frac{\sigma}{s}\,(a^2s\sigma+2AS)}-a\sigma\right),$$

$$\text{and } AD=\frac{s}{\sigma}BC.$$

NOTE 3. *Investigation of the preceding formulæ.* As the formulæ given in Note 2, have never been previously given by any author, it will be proper to

give their investigation here.—Let E be the point where A a, B b, would meet, if prolonged; (this point is not shewn in the figure), then $AE = \frac{as}{S}$, $BE = \frac{a\sigma}{S}$, area of the triangle $ABE = \frac{a^2 s\sigma}{2S}$, and the area of the triangle $CDE = \frac{a^2 s\sigma}{2S} - A$. Now because the triangles ABE, CDE, are similar,

$$\text{area } \triangle ABE : \text{area } \triangle CDE :: BE^2 : CE^2, \text{ that is,}$$

$$\frac{a^2 s\sigma}{2S} : \frac{a^2 s\sigma}{2S} - A :: \frac{a^2\sigma^2}{S^2} : CE^2, \text{ whence}$$

$$CE = \sqrt{\frac{2\sigma}{Ss}\left(\frac{a^2 s\sigma}{2S} - A\right)}, \text{ and}$$

$$BC = BE - CE = \frac{a\sigma}{S} - \sqrt{\frac{2\sigma}{Ss}\left(\frac{a^2 s\sigma}{2S} - A\right)}, \text{ or}$$

$$BC = \frac{1}{S}\left(a\sigma - \sqrt{\frac{\sigma}{s}(a^2 s\sigma - 2AS)}\right).$$

$$\text{Also } BE : BC :: AE : AD = \frac{AE}{BE} BC = \frac{s}{\sigma} BC.$$

The other modification of the formulæ for B C, when the point E falls on the other side of A B, is derived by making S negative.

SECTION II.

ON DIVIDING LANDS OR COMMONS, OF EITHER EQUAL OR VARIABLE VALUE, AMONG VARIOUS CLAIMANTS, IN PROPORTION TO THEIR SEVERAL CLAIMS.

Where land is the property of joint purchasers, co-heirs, co-partners, &c.; or where a common is to be divided among the adjoining proprietors in proportion to the values of their several estates; it becomes requisite to adopt the methods of division given in the following Problems, in conjunction with the Problems in the preceding Section. The present Section will, therefore, present cases of considerable complexity; but in order that these may be clearly understood, the simplest cases shall be first presented to the student.

PROBLEM I.

To divide a rectangular piece of ground of equal value throughout, either equally or unequally, among any given number of claimants, by fences parallel to one of its sides.

CASE I.—If the parts into which the rectangular space is to

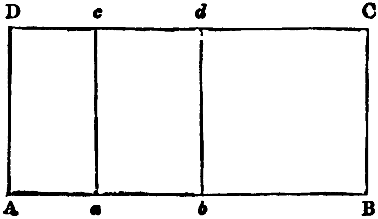
be divided be equal, it will be only required to divide two of the opposite sides of the rectangle into the given number of equal parts, and range the lines for the several fences to the consecutive points of division, and the land will then be divided as required. This appears to be too simple a case to require a diagram.

CASE II.—If the rectangular space is to be divided among several joint purchasers, who have paid unequal sums for the purchase thereof: then use the following

RULE.—As the sum of all the sums paid, is to the length of the side of the rectangle from which the division lines abut, so is any one person's sum to the breadth on the side of the rectangle due to that person; then divide both this side and the one opposite to it, by laying off the resulting breadths, and range lines to the corresponding points, and the rectangle will be divided by parallel lines, as required.

NOTE. This is obviously a question of single fellowship.

EXAMPLE.



Divide the rectangle A B C D, the length of which is 1470 links and its breadth 684 links, among three joint-purchasers P, Q and R, who paid for the purchase thereof respectively £120, £150, £220.

120£
150
220
——

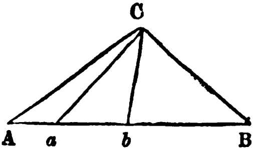
		links		£		links				
490	:	1470	: :	120	:	360	=	A a	=	P's breadth
——	:	——	: :	150	:	450	=	a b	=	Q's ditto
——	:	——	: :	220	:	660	=	b D	=	R's ditto
						——				

Whole breadth 1470 which proves the work.

NOTE. In this operation the breadth of the rectangle, which is common to all the three divisions, is not required to be used.

PROBLEM II.

To divide a triangle of equal value throughout, either equally or unequally, among several claimants, who shall all have the use of the same watering-place, situated at one of the angles of the triangular field.



EXAMPLE.

It is required to divide the triangular field A B C among three persons, whose claims therein are as the numbers 2, 3 and 5, so that they may all have the use of a watering-place situate at the angle C; A B being = 1000, A C = 685, and C B 610 links.

The rule in this Problem is the same as in the last.

$$\begin{aligned} \text{As } 2 + 3 + 5 = 10 : 1000 &:: 2 : 200 = A\,a, \\ \text{—} : \text{——} &:: 3 : 300 = a\,b, \\ \text{—} : \text{——} &:: 5 : 500 = b\,B; \end{aligned}$$

which are the portions of the base A B, belonging to the respective claimants; therefore, if lines be drawn from a and b to C, the triangular field will be divided in the required proportion, each claimant having the use of the watering-place at C.

NOTE. In solving this Problem it is not necessary to use the lengths of the sides A C, C B, because all the three triangles A C a, a C b, b C B have a common perpendicular; and, therefore, their areas are as their bases.

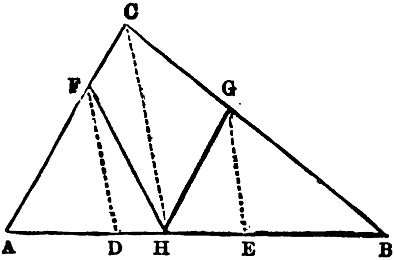
PROBLEM III.

To divide a triangular field of equal value throughout, either equally or unequally, among sundry claimants, by fences running from any given point in one of its sides.

The method of solving this Problem will be best shewn by the following

EXAMPLE.

Divide the triangular field ABC, the sides of which measure 30 chains = A B, 23 = B C, and 19 = A C, equally among three persons, by fences running from an occupation road that meets the side A B at H, which is 14 chains from A, that all the three persons may have the use of the road at H.



Divide AB into three equal parts in the points D, E; from H, (the point where the road meets A B) draw H C; parallel to which draw D F, E G meeting A C, B C respectively, in F and G; and join H F and H G, in which directions fences being made will divide the triangle as required.

NOTE. This method of solution is founded on the areas of triangles between the same parallels being equal: but it may be solved by actual measurement on the ground, by means of the following preliminary calculation.

ANOTHER METHOD.

As AH : AC :: AD $= \frac{1}{3}$AB : AF,
that is, 1400 : 1900 :: 1000 : $1357\frac{1}{7}$ links $=$ AF;
and HB : BC :: EB $= \frac{1}{3}$AB : BG,
that is, 1600 : 2300 :: 1000 : $1437\frac{1}{2}$ links $=$ BG.
whence the distances AF and BG may be measured from A and B; and from the points F and G the fences of division may then be drawn to H.

NOTE. When the claims of the several persons are unequal, it will be readily seen that AB is then only required to be divided in the proportion of the several claims, as in the preceding Problem, after which the solution will be the same, as that just given.

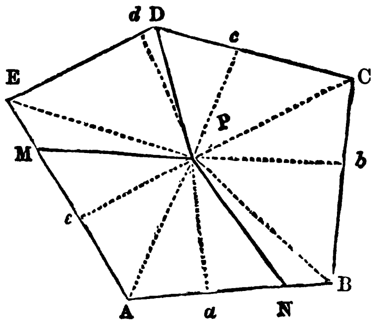
PROBLEM IV.

To divide an irregular field with any number of sides among sundry claimants, so that they may all have the use of a pond, situated at a given point within the field.

NOTE. The method of solving this Problem will be best seen from the method adopted in the following Example, in conjunction with the preceding Problems.

EXAMPLE.

Three persons Q, R, and S, bought a five sided field ABCDE, having a pond therein at P; for which they paid respectively £100, £150, and £200; it is required to divide the field into parts, in proportion to each person's claim, and so that each may have the use of the pond P, the quality of the land being equal throughout. The lengths of the sides and the perpendiculars on each of them to the centre of the pond P are as below, thus constituting five triangles.



AB $=$ 864	Pa $=$ 560
BC $=$ 827	Pb $=$ 608
CD $=$ 806	Pc $=$ 480
DE $=$ 682	Pd $=$ 544
EA $=$ 990	Pe $=$ 540

By multiplying eaah side by half the perpendicular thereon, the sum of the five products will be the area of the field; thus,

$$\begin{aligned}
864 \times 280 &= 241920 = \text{area of } APB,\\
827 \times 304 &= 251408 = \text{———} \ BPC,\\
806 \times 240 &= 193440 = \text{———} \ CPD,\\
682 \times 272 &= 185504 = \text{———} \ DPE,\\
990 \times 270 &= 267300 = \text{———} \ EPA.\\
\hline
\text{sum} &= 11{\cdot}39572 = \text{acres, the area of the field.}
\end{aligned}$$

The sums paid for the field by Q, R, and S, are as the numbers 2, 3, and 4, the sum of which is 9; therefore,

$$\begin{aligned}
9 : 11{\cdot}39572 &:: 2 : 2{\cdot}53238 = \text{Q's share,}\\
\text{—} : \text{———} &:: 3 : 3{\cdot}79857 = \text{R's do.}\\
\text{—} : \text{———} &:: 4 : 5{\cdot}06476 = \text{S's do.}
\end{aligned}$$

Let DP be assumed the divisional fence between Q and S's shares; then the area of the triangle DPE = 1·85504 acres is less than Q's share, therefore

$$\begin{aligned}
2{\cdot}53238 &= \text{Q's share,}\\
1{\cdot}85504 &= \text{area of } DPE,\\
\hline
67734 &= \text{difference.}
\end{aligned}$$

This difference must be taken from the triangle EPA to complete Q's share; this is best done by dividing the said difference by half the perpendicular P*e* of the triangle EPA, and the quotient will be the distance EM: thus,

$$\tfrac{1}{2} Pe = 270)\ 67734\ (250{\cdot}87 \text{ links} = EM.$$

The distance EM, being nearly 251 links must now be measured from E on EA, which will give the point M, and, a straight fence being set out from M to P, will cut off Q's share.

The remainder of the triangle EPA, viz. 2·67300 − ·67734 = 1·99566 is less than R's share, therefore,

$$\begin{aligned}
3{\cdot}79857 &= \text{R's share,}\\
1{\cdot}99566 &= \text{area } MPA,\\
\hline
1{\cdot}80291 &= \text{difference.}
\end{aligned}$$

This difference must be taken from the triangle APB to complete R's share, as before: thus,

$$\tfrac{1}{2} Pa = 280)\ 1\ 80291\ (643{\cdot}9 \text{ links} = AN.$$

This distance being measured from A towards B, will give the point N; and, the fence NP being now set out, will divide

the field as required; the triangles N P B, B P C, C P D making up the exact quantity required for S's share, as may be readily shewn by adding their three areas together, which will prove the accuracy of the work: thus,

	864	= A B
	643·9	= A N
difference	220·1	= N B
	280	= $\frac{1}{2}$ P *a*
	61628·0	= area N P B
	251408	= —— P B C
	193440	= —— C P D
sum	5·06476	= S's share, which proves the work.

NOTE 1. If some or all of the fences of the field A B C D E had been crooked, the operation of division would have been the same, excepting that the quantities of the offsets would have to be taken into the account, thus making a little additional calculation. It will at once be seen that this method of division may be extended to any number of claimants, whatever be the shape of the ground to be divided, the dotted lines from P to the angles of the field not requiring to be measured.

NOTE 2. All the surveyors, who have written on the subject of this Problem, use what they call "*guess lines*" to effect the division, and correct the resulting errors by dividing the double quantities in excess or defect by these "guess lines" for the perpendiculars to determine the correct positions of the divisional fences. The student will at once perceive that this is a circuitous, blundering, and unscientific method of proceeding, and should be avoided in every case, except where the boundary is very crooked, and the divisional fences are not required to be continuous straight lines.

NOTE 3. In this Problem the division is effected and proved without the aid of a plan, but it would, perhaps, be better for the satisfaction of the claimants, as well as for the surveyor himself to plan the whole of the work; especially as an error in the work might thus be more readily detected.

PROBLEM V.

To set out from a field or common of variable value, a quantity of land, that shall have a given value, by a straight fence in a given direction.

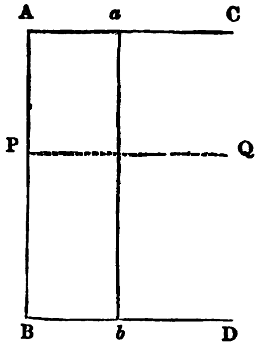
NOTE. This Problem presents a great variety of cases, the most simple of which shall be first produced.

CASE I.—C A B D is a portion of a straight-sided field, right angled at A and B, and P Q a right line, also at right angles to A B, dividing the field into portions of different values; it is required to lay off a quantity of land *a* A B *b* of given value, by a straight fence *a b* parallel to A B.

N.B. Lines, such as P Q, are technically called quality lines.

Rule.—Multiply A P, PB, respectively, by the value of the adjoining land, take the sum of the products, divide the value of the whole portion to be laid off with five cyphers annexed by this sum, and the quotient will be the required breadth A *a* or B *b*.

EXAMPLE.



Let A P = 500, P B = 700 links; the value of the land between A C and P Q £50, and that between P Q and B D £80 per acre; it is required to lay out land to the value of £300, by a straight fence *a b* parallel to A B.

$$\begin{array}{lrl} \text{Here} & 500 \times 50 = & 25000 \\ & 700 \times 80 = & 56000 \\ \hline & & 81000 = \text{sum of products; and} \end{array}$$

$$81{\cdot}000 \left\{ \begin{array}{l} 9)30000{\cdot}000 = \text{£300 with 5 cyphers annexed,} \\ \hline 9)3333{\cdot}3' \\ \hline 370{\cdot}37 = 370\tfrac{1}{3} \text{ links nearly} = \mathrm{A}a = \mathrm{B}b. \end{array} \right.$$

Note 1. If the "quality line" P Q be not perpendicular to A B, it may be made so by "giving and taking;" especially as the required breadth A *a* may be nearly known by a rough calculation: besides, this liberty with the quality line is usually admitted, since the exact boundary of different qualities of lands cannot be accurately defined.

Note 2. This Problem is of great importance, as land of variable quality is very frequently required to be laid out, in the enclosure of extensive commons, by straight fences in given directions for the purposes either of drainage or of irrigation. If any crooked portion of the land to be inclosed, lie to the left of A B, it must be measured, valued, and deducted from the whole value of the land to be laid out; then the Rule, here given, may be applied to the remaining value.

Note 3. If the three parallel straight lines A *a*, B *b*, P Q, are not at the same time perpendicular to A B, the Rule just given, will equally apply, excepting that the distance of *a b* from A B must be laid off perpendicularly to A B.

Note 4. This rule for laying out land of variable quality has not, to my knowledge, been given by any previous author, the method of performing so simple an operation having been invariably by approximating to the true quantity by means of "guess lines."——*Investigation of the Rule.* Put A P = *a*, P B = *b*, A *a* = B *b* = *x*, *m* = value of land above P Q, *n* = value of land

below it, V = whole value to be laid out, and l = square links in one acre: then $\frac{amx}{l}$ = value of land adjoining AP and $\frac{bnx}{l}$ = value adjoining PB;

$$\text{therefore } \frac{amx}{l} + \frac{bnx}{l} = V; \text{ whence}$$

$$x = \frac{lV}{am + bn}, \text{ Q. E. I.}$$

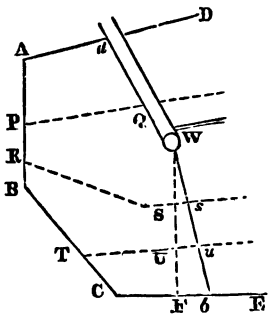
This rule may be obviously extended to any number of different qualities of land, bounded by parallel quality lines, by multiplying the breadth of each quality by its price, and taking the sum of all the products for a divisor.

CASE II.—When the divisional line is required to pass through a given point W, where an occupation road and watering place is situated, which is to be used by the owners of three or more shares of a common of variable value.

The method of proceeding in this case will be best understood from the following

EXAMPLE.

Let D A B C E be a portion of a common of variable value, W a watering place, and dW a road, forming part of the divisional line: it is required to set out a quantity of land of a given value V by a straight fence W b; m being = value of land between PQ and RSs, n = value between R S s and T u, o = value between T u and C E, and p = value between PQ and Ad, the quantity of which is given by boundaries already determined.



First draw W F perpendicular to C E, and find the value of the land to the left of a W F, let this value = v which in this case is assumed to be less than the given value V; therefore, the quantity of land required to the right of W F will be V − v, which put = V′, and having adjusted the portions of the quality lines S s, U u perpendicular to W F, as per Note 1, Case I., let W S = a, S U = b, U F = c, their sum, or the whole distance, W F = s, and F b = x, the symbols denoting the different values of the land being given in the example, and l = square links in an acres: then

$$x = \frac{2lsV'}{a^2m + (2a + b)bn + (2a + 2b + c)co} = Fb;$$

which distance being measured, the straight fence W b may now be set out, and the other shares to the right of d W b may be next proceeded with, according to the method given in this or the following cases.

THE ABOVE EXAMPLE IN NUMBERS.

W S $= a = 460$, S U $= b = 400$, U F $= c = 420$ links; the values are $m =$ £30, $n =$ £40, and $o =$ £60 per acre; and the value V$'$ = £80: required the distance F b or x by the preceding formula.

Here $s = a + b + c = 460 + 400 + 420 = 1280$ links, whence

$$x = \frac{2 \times 1280 \times 100000 \times 80}{460^2 \times 30 + (920 + 400) \times 400 \times 40 + (920 + 800 + 420) \times 420 \times 60}$$

$$= \frac{20480000 \cdot 000}{81396 \cdot 000} = 251 \cdot 6 \text{ links} = \text{F } b,$$

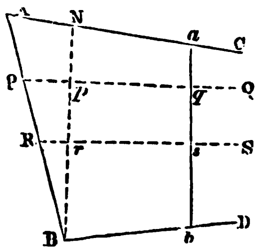
which determines the position of the fence W b.

NOTE. This method possesses the advantage of being practicable on the ground without the help of either a map or guess-line; however, in the case of the division of commons a map is always necessary for the satisfaction of the several parties interested therein.—*The investigation of formula* here given, is founded on similar triangles combined with the same principle as that in Case I. If there be less than three different qualities of land to be laid out, the symbols referring to the additional qualities, must be made to vanish in the formula; and, if there be more than three qualities to be laid out the law for the extension of the formula is obvious.

CASE III.—To set out from a field or common of any form, and of variable value, a quantity of land of given value by a straight fence in a given direction.

EXAMPLE.

Let C A B D be a portion of a common, of variable value, from which it is required to set out a quantity of land of a given value V, by a straight fence $a\,b$ parallel to B N, the quality lines P Q, R S being so adjusted as to be perpendicular to B N at p and r respectively, as per Note 1, Case I., and the value of the land between A C, P Q being $= m$, between P Q, R S $= n$, and between R S, B D $= o$.



First find the value of the land in the triangle A N B, and if the fence A B had been crooked, the offsets would have to be included; the value of the land to the left of B N being supposed to be less than

is required to make up the given value V. Let the value of the land in the triangle A N B $= v$, then the value of the land still remaining to be set out will be $V - v$, which put $= V'$, let $Np = a$, $pr = b$, $rB = c$, $pq = rs = x$, cot. of the angle $CNB = 2\alpha$, and cot. of the angle $DBN = 2\beta$, the symbols denoting the different values of the land being already given in the example, and $l =$ square links in an acre; then

$$x = \frac{2lV'}{am + bn + co\sqrt{(am + bn + co)^2 - 4(am + \beta o)lV'}} = pq \text{ or } rs;$$

which distance may be set out at any two points perpendicular to N B; and, if the land in question be a common, requiring the shares of several claimants to be set out, the quality lines may be again adjusted to the right of *a b*, and the next share may then be set out as before.

NOTE 1. The investigation of the general formula, just given, is the same in principle as those in the preceding cases, being only a little more complex, on account of the land to be set out not being rectangular, and therefore involving the solution of a quadratic equation, thus giving an *apparent* complexity to the result which will doubtless startle the generality of surveyors, who are accustomed to use "*guess lines*" when even the most trifling difficulty occurs, thus making repeated approximations to obtain the position of the correct divisional fences, which may be obtained at once by these methods, with even less calculation than by guess lines, without naming the repeated measurements accompanying these bungling operations. It would, therefore, appear that neither the surveyors, nor the writers on this department of surveying, have had the least acquaintance with Mathematics, otherwise the results here given, which are comparatively void of extreme complexity, would have been known long ago.

NOTE 2. When one or both of the angles C N B, D B N are obtuse, their ½ cotangents α and β will be one or both negative respectively; and when these angles are right ones, their cotangents vanish; and the formula for the perpendicular breadth pq or rs becomes.

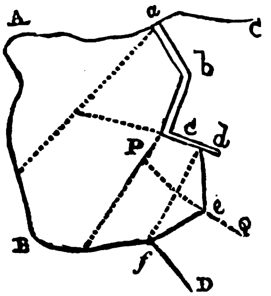
$$x = \frac{lV'}{am + bn + co} = pq = rs.$$

which is the same as the formula given in Case I., as it obviously ought to be.

CASE IV.—When the boundary of the land to be laid out is very irregular, also when part of the divisional fence is predetermined, either for the purpose of drainage or irrigation, and when the quality lines run irregularly.

EXAMPLE.

Let C A B D be the boundary of a portion of common, from which it is required to lay out a portion of land of variable value, part of the divisional fence of which it is desirable to have in the direction of a watercourse *abcd*, and the remainder of the divisional fence to run from *d* to *f*.



Join df, and find the value of the land included thereby and the other boundaries in question, and, this value being found less than the required value, the remaining land still required may be determined by assuming two lines, so as to form a triangle on df, as a base, till the correct position of the remaining part def of the divisional fence shall be ascertained, it being advisable to let the angle e, in the required fence, fall on the quality line P Q, for the purpose of more readily calculating the areas of the two triangles formed thereby, and from thence finding their values.

NOTE. It will here be proper to notice that surveyors who are unaccustomed to use mathematical formula, will more readily and accurately make the divisions of commons, &c., by means of assumed or guess lines, than by the methods I have given in the three first Cases of this Problem. Their not having had any practice beyond finding the areas of triangles, &c., gives them little confidence in more scientific methods, which they therefore view as chimeras, or at best as methods very difficult to be understood. It is to be hoped that the very general diffusion of science at the present day will root out this ignorant prejudice.

PROBLEM VI.

CASE I.—*To divide a common of uniform value among any number of proprietors, in proportion to the values of their respective estates.*

In this case, the quantities and values per acre of each proprietor's estate must be determined, by survey if necessary, and the map, with the quantity of the common, must also be prepared; the value of the common, being uniform, is not required.

TO FIND EACH PROPRIETOR'S SHARE.

RULE.—As the sum of the values of each person's estate : the whole quantity of common to be divided : : the value of each person's estate : to that person's share.

EXAMPLE.

Divide a common of 100 acres among three persons, A, B, and C; A's estate is 120 acres at £4 per acre, B's is 180 acres at £5 per acre, and C's 260 acres at £2 per acre.

£	£									
120 × 4 =	480									
180 × 5 =	900									
260 × 2 =	520									
	——	acres				*a.*	*r.*	*p.*		
	1900 :	100	::	480 :		25	1	2·1	=	A's share
	—— :	——	::	900 :		47	1	18·9	=	B's do.
	—— :	——	::	520 :		27	1	18·9	=	C's do.
						99	3	39·9		proof.

The quantity of each person's share being now determined, the common may be divided by one or other of the methods already given.

CASE II.—*To divide a common of variable value among any number of claimants, in proportion to the values of their respective estates.*

In this case, the map of the common with the quantity of every different quality of land thereon, must be prepared, as well as the quantities and qualities of the several parts of each claimant's estate, in order that the values of both the common and each person's estate may be properly ascertained; the quality lines on the common being marked out by the valuer or valuers, previous to the survey being made.

When the survey of the common is completed and mapped, with the quality lines laid down thereon, the several qualities must be number 1, 2, 3, &c., with the values of the land corresponding to each number, as determined by the valuers, who are usually called "Commissioners." Most surveyors use the letters of the alphabet to denote the different values of the land: thus *a* may represent five shillings, *b* six shillings, &c., per acre per annum. By using the letters in this manner a multiplicity of figures, which may be confounded with other numbers, is avoided.

ROADS, QUARRIES, WATERING PLACES, &c., REQUIRED TO BE SET OUT PREVIOUS TO ENCLOSING COMMONS, WASTES, &c.

Before the lands of a common or waste can be divided and allotted, both public and occupation roads must be set out in the most convenient manner; they should be straight and, as far as practicable, at right angles to one another, as this arrangement not only facilitates the division of the land, but contributes greatly to the economy of cultivation with the plough. All old roads that may be deemed unnecessary may be stopped up and

allotted to the different claimants, or diverted into more convenient directions, at the discretion of the Commissioners.

Portions of the common are now to be set apart for quarries, sand or gravel pits, if such abound in the common. The ground, thus set out, is considered as the common property of the several claimants, for the purpose of building, making roads, &c. Also, if there are any good springs or ponds on the common, they must be left unenclosed, in like manner, for common use; or the water must be conveyed from them by drains or channels to more convenient situations, previous to the enclosure of the common.

The lord of the manor in some places claims $\frac{1}{12}$th of the common, in some $\frac{1}{16}$th, &c. His claim, whatever it be, must be next set out, after its value has been determined from the whole value of the common. The lord of the manor will also be further entitled to his share of common, in proportion to his property, in the same manner as the other proprietors.

Lastly, when the roads, watering places, quarries, sand and gravel pits, and manorial rights have been set out, the remainder of the common must be divided equitably, as it respects quantity, quality, and situation, among the proprietors of lands, tenements, houses, cottages, &c., situated in the parish or township where the inclosure is to be made.

THE METHOD OF DIVIDING AND ALLOTTING THE REMAINING PART OF THE COMMON.

Having found the sum of the values of each proprietor's estate, and the whole value of the remaining part of the common to be divided, proceed to find the value of each proprietor's share by the following

RULE.—As the sum of the value of each proprietor's estate, is to the whole value of the common remaining to be divided, so is the value of each proprietor's estate to his share of the value of the common.

It will be quite unnecessary to give an example in this case, as the laying out of the shares of the several claimants, after their respective values have been found by the above Rule, would only be a repetition of the methods already given, on a large scale; and after the work of laying out the several inclosures on the ground has been completed, the last enclosure or share of the common must be of the same value as that assigned by the rule, or so very near to it that the error is of no importance; otherwise a mistake has been made which must be immediately rectified.

For further information respecting the law for the inclosure of commons, waste lands, &c., the student may consult the following

ACTS OF PARLIAMENTS.

GENERAL ACT.

"An Act for consolidating in one Act, certain provisions usually inserted in Acts of Inclosure; and for facilitating the mode of proving the several facts, usually required on the passing of such Acts." (Act. 41 Geo. III., chap. 109. 1801.)

Since the General Act, the following Acts have been passed, relating to inclosures.

1. "An Act to amend the Law respecting the inclosing of open fields, pastures, moors, commons, and waste lands in England." (Act 1 & 2 Geo. IV. chap. 23; April 19th, 1821.)

2. "An Act for facilitating the inclosure of open and arable fields in England and Wales." (Act 6 & 7 Will. IV. chap. 115; August 20th, 1836.)

3. "An Act to extend the Powers and Provisions of the several Acts relating to the enclosure of open and arable fields in England and Wales." (Act 3 & 4 Vict. chap. 31; July 23rd, 1840.)

4. "An Act to facilitate the enclosure and improvement of commons and lands held in common, the exchange of lands, and the division of intermixed lands; to provide remedies for defective or incomplete Executions, and for the non-execution of the Powers of general and local Inclosure Acts; and to provide for the revival of such Powers in certain cases." (Act 8 & 9 Vict. chap. 118; August 8th, 1845.)

5. "An Act to amend the Act to facilitate the inclosure and improvement of commons." (Act 9 & 10 Vict. chap. 70; August 26th, 1846.)

NOTE. All the above Acts may be procured through any bookseller in town or country.

ENGINEERING SURVEYING.

PART II.

LEVELLING.

CHAPTER I.

DEFINITION OF LEVELLING.

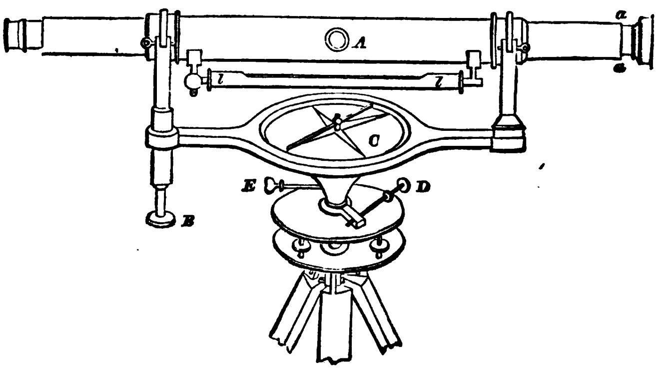
By the art of levelling the inequalities of the upper boundary of any section of the earth's surface may be shewn, and thence may be determined the several heights of any number of points in that boundary, above or below an assumed line, called a level line; though, in reality, this line is a great circle of the earth, and is such as would be derived from a section of the surface of still water.

LEVELLING INSTRUMENTS.

(1.) Levelling instruments all depend on the action of gravity: of these the plumb-line, on which the mason's level depends, is the most simple; but it cannot be used in extensive operations, on account of its practical inconvenience. The fluid, or water level, in all its modifications, is also found inconvenient for extensive practice.

(2.) Spirit levels are now commonly used, as the most accurate instruments for finding the differences of level, or vertical distances between two stations: of these there are three, which we shall proceed to describe, namely, the Y level, Troughton's, and Gravatt's levels.

THE Y LEVEL.



The foregoing figure represents this instrument. A is an achromatic telescope, resting on two supporters, which in shape resemble the letter Y; hence the name of the instrument. The lower ends of these supporters are let perpendicularly into a strong brass bar, which carries a compass box C. This compass is convenient for taking bearings, and has a contrivance for throwing the needle off its centre, when not in use. One of the Y supporters is fitted into a socket, and can be raised or lowered by the screw B.

Beneath the compass box, which is generally of one piece with the bar, is a conical axis passing through the upper of two parallel plates, and terminating in a ball supported by a socket. Immediately above the upper parallel plate is a collar, which can be made to embrace the conical axis tightly by turning the clamping screw E; and a slow horizontal motion can be given to the instrument by means of the tangent screw D. The two parallel plates are connected together by the ball and socket already mentioned, and are set firm by four mill-headed screws, which turn in sockets fixed to the lower plate, while their heads press against the under side of the upper plate, and thus serve the purpose of setting the instrument truly level.

Beneath the lower parallel plate is a female screw, adapted to the staff head, which is connected with brass joints to three mahogany legs, of exactly the same construction as those already described for supporting the theodolite.

The spirit level *l l* is fixed to the telescope by a joint at one end, and a capstan-headed screw at the other, to raise or depress it for adjustment.

(3.) Previous to using this instrument the following adjustments must be attended to.

1. *The Adjustments of the telescope for parallax and collimation.*

2. *The adjustment of the bubble tube.*

3. *The adjustment of the axis of the telescope perpendicularly to the vertical axis.*

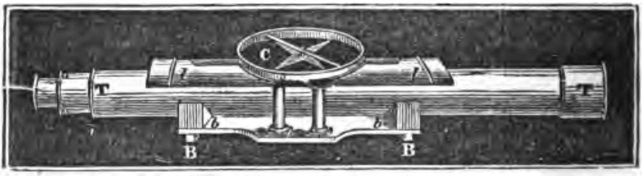
1. *The adjustment for parallax and collimation* have already been described, (p. 76,) being the same as those required for the theodolite.

2. *The adjustment of the bubble tube.*—Move the telescope till it lies in the direction of two of the parallel plate screws, and by giving motion to these screws bring the air-bubble to the centre of its run. Now reverse the telescope carefully in the Ys, that is, change the places of its ends; and should the bubble not settle in the same point of the tube as before, it

shews that the bubble tube is out of adjustment, and requires correcting. The end to which the bubble retires must then be noticed, and the bubble made to return one half the distance by turning the parallel plate screws, and the other half by turning the capstan-headed screws at the end of the bubble tube. The telescope must now again be reversed, and the operation repeated, until the bubble settles at the same point of the tube, in the centre of its run, in both positions of the instrument. The adjustment is then perfect, and the clips that confine the telescope in the Ys should be made fast.

3. *The adjustment of the axis of the telescope perpendicularly to the vertical axis.*—Place the telescope over two of the parallel plate screws, and move them, unscrewing one while screwing up the other, until the bubble of the level settles in the centre of its run; then turn the instrument half round on its vertical axis, so that the contrary ends of the telescope may be over the same two screws, and, if the bubble does not again settle in the same point as before, half the error must be corrected by turning the screw B, and the other half by turning the two parallel plate screws, over which the telescope is placed. Next turn the telescope a quarter round, that it may be over the other two screws, and repeat the same process with these two screws; and when, after a few trials, the bubble maintains the same position in the centre of its run, while the telescope is turned round on the vertical axis, this axis will be truly vertical; and the axis of the telescope being horizontal, by reason of the previous adjustment of the bubble tube, will be perpendicular to the vertical axis, and remain truly horizontal, while the telescope is turned completely round. The adjustment is therefore perfect.

TROUGHTON'S LEVEL.



(4.) In this level the telescope T rests close down upon the horizontal bar *b b*, the spirit level *l l* is permanently fixed to the top of the telescope, and does not therefore admit of adjustment; and the compass box C is supported above the level by four small pillars, attached to the horizontal bar. This construction makes the instrument very firm and compact. Its parallel

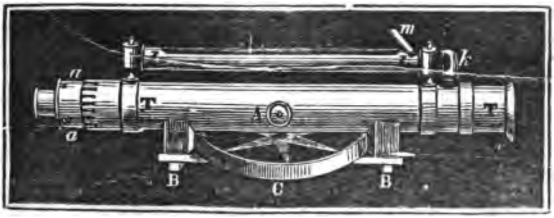
plates and supporting parts are the same as those already described for the theodolite.

The diaphragm is furnished with three threads, two of them vertical, between which the levelling staff may be seen, and the third, horizontal, gives the reading of the staff by its coincidence with one of the graduations marked upon it. Sometimes a pearl micrometer scale is fixed on the diaphragm, instead of the wires. The central division on the scale, in this case, indicates the collimating point, and by its coincidence with a division of the levelling staff gives the required reading on the staff; and the scale serves the purpose of measuring distances approximately, and of determining stations nearly equidistant from the instrument, since at such equal distances the staff will subtend the same number of divisions on the micrometer scale.

The line of collimation is set perpendicular to the vertical axis, in the same manner as in the theodolite already described.

This level is preferred by many, on account of its adjustments not being likely, after they are once perfected, to become deranged.

GRAVATT'S LEVEL.

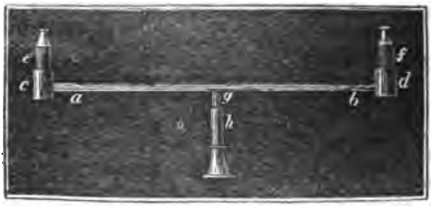


(5.) This instrument is furnished with an object-glass of large aperture, and short focal length; and, sufficient light being thus obtained to admit of a higher magnifying power in the eye-glass, the advantages of a much larger instrument are secured, without the inconvenience of its length:—it is hence called "*the dumpy.*" The diaphragm is carried by the internal tube *a a*, which is nearly equal in length to the external tube. The external tube T T is sprung at its aperture, and gives a steady and even motion to the internal tube *a a*, which is thrust out, and drawn in, to adjust the focus to objects at different distances by means of the milled headed screw A. The spirit level is placed above the telescope, and attached to it by capstan-headed screws, one at each end, by means of which the bubble can be brought to the centre of its run, as in the case of

the Y level, already described. The telescope is attached to a horizontal bar in a similar manner as Troughton's level, but room is just left between the telescope and bar for the compass box. A cross level *k* is placed upon the telescope at right angles to the principal level *l l*, by which we are enabled to set up the instrument at once nearly vertical. A mirror *m* mounted upon a hinge-joint is placed at the end of the level *l l*, so that the observer, while reading the staff, can at the same time see that the instrument retains its proper position—a precaution by no means unnecessary in windy weather, or on soft spongy ground.

The telescope is attached to the main bar by capstan-headed screws B B, as in Troughton's level, by which the line of collimation is set perpendicular to the vertical axis; and the instrument is set upon parallel plates, &c., like the theodolite and the levels already described. This level is much preferred and used by many engineers.

(6.) After having treated of the more perfect levelling instruments, it will now be proper to describe the *water level*, a very simple instrument, adapted to give a rapid delineation of any section of the earth's surface, where very strict accuracy is not required. It can be made by any workman, will cost but a few shillings, and requires no adjustment when using it. *It is greatly to be recommended to farmers for determining the levels for draining their lands.*



"*a b* is a hollow tube of brass, about half an inch in diameter, and about three feet long; *c* and *d* are short pieces of brass tube of larger diameter, into which the long tube is soldered, and are for the purpose of receiving the two small bottles *e* and *f*, the ends of which, after the bottoms have been cut off, by tying a piece of string round them when heated, are fixed in their positions by putty or white lead; the projecting short axis *g* works in a hollow brass cylinder *h*, which forms the top of a stand: but it may be made in a variety of ways, so as to revolve on any light portable stand. The tube when required

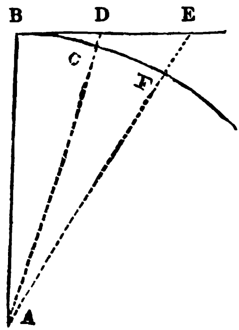
for use, is filled with water, coloured with lake or indigo, till it nearly reaches the necks of the bottles, which are then corked for the convenience of carriage. On setting the stand tolerably level with the eye, these corks are both withdrawn, which must be done carefully, and when the tube is nearly level, otherwise the water will be ejected from one of the bottles; and the surface of the water in the bottles, being necessarily on the same level, gives a horizontal line in whatever direction the tube is turned, by which the vane of a levelling staff may be adjusted."

LEVELLING STAVES.

6.) The best constructed levelling staff (Gravatt's) consists of three parts sliding one within another, and, when opened out for use, forms a staff 17 feet long, jointed together something after the manner of a fishing rod. The whole length is divided into hundredths of a foot, alternately coloured black and white, and occupying half the breadth of the staff; but for distinctness the lines denoting tenths of feet are continued the whole breadth, every half foot or five tenths being distinguished by a conspicuous black dot on each side, the whole feet being numbered with the figures 1, 2, 3, &c.

CORRECTION FOR CURVATURE.

(7.) Let B D E be a horizontal line, that is, such as would be given by the line of sight of a level, properly adjusted; B C F an arc of a great circle of the earth, and A its centre.



It will at once appear from the figure, that the heights DC, EF, of the apparent level B E, above the true level increase successively from the point B. The height EF of the apparent level above the true, is equal to the square of the distance B E divided by twice the earth's radius A B, that is $EF = \frac{BE^2}{2AB}$,* and similarly $DC = \frac{BD^2}{2AB}$, &c., therefore the corrections for curvature, D C, E F, &c., vary as the squares of the distances BD, BE, &c., since 2AB is a constant quantity.

Taking the earth's radius to be 3956½ miles, and assuming the

* The demonstration of this formula is given in my edition of Nesbit's Surveying, p. 348.

distance B D to be 1 mile, then the correction for curvature $DC = BD^2 \div 2AB = 1^2 \div 7918 = \frac{1}{7918}$ of a mile $= 8{\cdot}007$ inches = nearly 8 inches. If the distance $BE = 3$ miles, then the correction $EF = BE^2 \div 2AB = \frac{9}{7918} = 72{\cdot}0637$ inches, or more than 6 feet.

Let any distance $BD = d$ in miles, and the correction for curvature for 1 mile be taken = 8 inches = $\frac{2}{3}$ of a foot, which it is very nearly; then

$$\text{correction} = \frac{2\,d^2}{3} \text{ feet,}$$

for any distance d in miles:

and let $\frac{c}{80} = d$, c being chains; then

$$\text{correction} = \frac{2\,d^2}{3} = \frac{2 \times 12\,c^2}{3 \times 80^2} = \frac{c^2}{800} \text{ inches,}$$

for any distance c in chains.

CORRECTION FOR REFRACTION.

(8.) The effect of the earth's curvature is modified by another cause, arising from optical deception, namely, refraction; the correction for which varies with the state of the atmosphere, but it may generally be taken at $\frac{1}{7}$ of the correction for curvature, as an average; and since refraction makes objects appear higher than they really are, the correction for it must be deducted from that for curvature.

EXAMPLES.

1. Required the correction for curvature and refraction, when the distance of the object is $2\frac{1}{2}$ miles.

$$\frac{2}{3} \times (2{\cdot}5^2) = \frac{2 \times 6{\cdot}25}{3} = 4{\cdot}166 \text{ cor. for curvature.}$$

$\frac{1}{7}$ of which is.................. ·595 cor. for refraction.

Difference..................... 3·571 feet, cor. required.

2. Required the correction, as in the last example, when the distance is 60 chains.

$60^2 \div 800 = 4.5$ cor. for curvature

$\frac{1}{7}$ of which is ·643 cor. for refraction.

Difference ... 3·857 inches, cor. required.

3. From a point in the Folkstone road, the top of the keep of Dover Castle was observed to coincide with the horizontal wire of a levelling telescope, when adjusted for observation, and therefore was apparently on the same level; the distance of the instrument from the castle was $4\frac{1}{2}$ miles, required the correction

for curvature and refraction, that is, the true height of the keep of the castle above the point of observation.

$$\frac{2}{3} \times (4{\cdot}5)^2 = \frac{40{\cdot}5}{3} = 13{\cdot}5 \text{ feet, cor. for curvature.}$$

$\frac{1}{7}$ of which .... $=$ 1·93 feet, cor. for refraction.

Difference .... $=$ 11·57 feet, cor. required.

See also the tables for these corrections at the end of the book.

PRINCIPLES AND PRACTICE OF LEVELLING.

To find the differences of the levels of several points on the surface of the earth.

(9.) Before entering on this subject, it will be proper to state that the corrections for curvature and refraction, already explained, are seldom applied in the practice of levelling, the spirit level being usually placed midway between the stations, the levels of which are to be observed, hence the resulting corrections for each station are equal, and therefore the difference of the levels at the two stations is as truly shewn by the difference of the readings of the two staves fixed thereon, as if the corrections had been made. Thus the trouble of making these corrections is avoided by *simply placing the instrument midway between the two staves.*

(10.) Let it be required to find the difference of level between the points A and G.—A levelling staff is erected at A, the instrument is set up and adjusted at B, another staff is also erected at C, at the same distance from B that B is from A, as nearly as can be judged by the eye; the readings of the two staves are

then noted; the horizontal lines, connecting the staves with the instrument, represent the visual ray or level line of sight. The instrument is then conveyed to D, and the staff that stood at A is now removed to E, the staff C retaining its former position, only its graduated side turned to the instrument, and from being the fore staff at the last observation, it is now the back staff: the reading of the two staves are again noted, and the instrument removed to F, and the staff C to the point G,

the staff at E retaining its position, now in its turn becomes the back staff, and so on to the end of the work, which may thus be continued to any extent. The difference of the readings of the staves at A and C will shew the difference of level between the points or stations A and C, because the visual line of the instrument is virtually level, and the same is true with respect to every two consecutive stations.

EXAMPLE.

Back sight on staff A	10·66 feet.
Fore sight on staff C	11·78
The fall from A to C	1·12 difference.

Because when the front reading is the greater the ground falls and *vice versa.*

Back sight on staff C	13·36
Fore sight on staff E	9·16
The rise from C to E	4·20 difference.
Subtract the fall from A to C...	1·12
The rise from A to E	3·08 difference.

Because the rise from C to E is greater than the fall from A to C, their difference shews the total rise.

Back sight on staff E	7·62
Fore sight on staff G	8·16
The fall from E to G	0·54 difference.

This fall taken from the rise from A to E, that is,

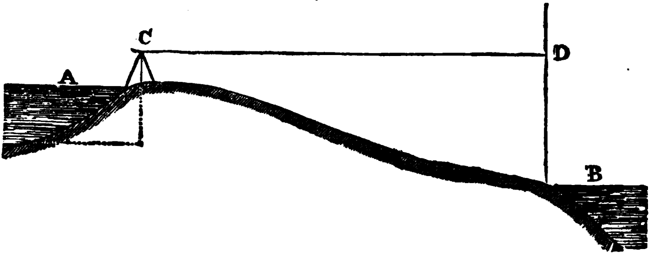
3·08
0·54

gives the total rise from A to G .... 2·54, or nearly 2 feet $6\frac{1}{2}$ inches.

The difference of the sums of the back and fore readings of the staves, will more readily give the difference of level between A and G: thus,

	Back sights.	Fore sights.
	feet.	feet.
	10·66 at A	11·78 at C
	13·36 at C	9·16 at E
	7·62 at E	8·16 at G
sums	31·64	29·10
	29·10	
	2·54 difference of level, the same as before.	

(11.) The following is an example of levelling performed at only one operation, and will therefore require the correction for curvature and refraction.



It is required to drain a pond by making a cut to a stream at B, at a distance of 60 chains: let a level be set up and adjusted at C, and then directed to a staff, held upright, at the edge of the water at B. The horizontal line C D represents the line of sight, cutting the staff at D, the reading being 15·84; the height of the instrument above the ground was 4·8 feet, and the depth of the pond 10 feet: what is the difference of level between the bottom of the pond and the surface of the stream?

		ft. dec.
Reading of the staff		15·84
Height of the instrument	4·80	
Depth of pond	10·00	
Correction for curvature and refraction for 60 chains—(see tables at the end of the book)	0·32	
		15·12
Difference of level		0·72

which is little more than 8½ inches, and just a sufficient fall to make the water run freely from the bottom of the pond to the surface of the stream at B; it having been found in practice that a less amount of descent than from 8 to 12 inches per mile produces no efficient current for the purpose of drainage.

TO DRAW A SECTIONAL LINE OF SEVERAL POINTS IN THE EARTH'S SURFACE, THE LEVELS OF WHICH HAVE BEEN TAKEN (fig. p. 138).

(12.) Let *a*, *b*, *c*, *d*, *e*, *f*, and *g* be the several points; then, in order to draw the section to shew the undulations of the ground between *a* and *g*, the distances of the several points from *a*, in

addition to their levels, must be taken; this is usually done during the operation of levelling. These distances, with the back and fore sights, may be arranged in a level book of the following form, which, though not the form practically used, will probably be more clearly understood.

LEVEL BOOK.

Back Sights.	Fore Sights.	Fall.	Rise.	Reduced Levels.	Distances in Chains, and Remarks.
3·50	5·65	2·15		2·15	4·60 at *b* on road.
4·10	10·85	6·75		8·90	7·80 at *c*.
5·04	9·25	4·21		13·11	11·60 at *d*.
3·84	12·91	9·07		22·18	15·20 at *e*.
4·12	7·65	3·53		25·71	bottom of canal, distance 2·16
10·49	3·92		6·57	19·14	21·00 at *f*.
12·96	3·03		9·93	9·21	27·00 at *g*.
44·05	53·26				
	44·05				
diff.	9·21	the same as the last of the reduced levels.			

In this level book it will be seen that the differences 2·15 and 6·75, in the column marked Fall, are added together, making 8·90, thus giving the fall at *c*, in the column marked Reduced Levels: to this sum the succeeding falls are added, one by one, till we get the fall 25·71 at the bottom of the canal, which is the lowest point. Then the differences in the column marked Rise, are subtracted successively from 25·71 for the falls at *f* and *g*; the latter of which is 9·21, the total fall from *a* to *g*, which, agreeing with the difference of the sums of the back and fore sights, shews the truth of the castings. The last column shews the distances of the several points *b*, *c*, &c., from *a*, in chains, with other remarks.

DATUM LINE.

(13.) The section might be plotted by laying off the distances in the last column in the preceding level book on a horizontal line, and setting off their corresponding numbers of feet, in the column marked Reduced Levels, perpendicularly below the line; but it is found inconvenient in practice to plot a section in all cases after this method, as in extensive operations the reduced levels would repeatedly fall above and below the line in ques-

tion, and thus confuse the operation; therefore a line AG called "the datum line" is assumed at 100, 200 feet, &c., below the first station *a*; thus making that line always below the sectional line *af*, of which a clearer view may be obtained.

(14.) In the following practical level book the rise or fall is respectively added to or subtracted from the assumed distance of the datum line, and the next rise or fall, again added to or subtracted from the sum or difference:—thus 2·15, being a fall, is subtracted from 100 (the assumed distance of the datum line) leaving 97·85 feet, the height of the ground at *b*: the next fall 6·75 is then subtracted from 97·85, leaving 91·10 feet for the height at *c*; and so on to 3·53, which is the last fall:—the next 6·57, being a rise, is added, as well as 9·93;—thus the last reduced level is 90·79 feet, which taken from the datum 100 leaves 9·21 feet, agreeing with the differences of the sums of the back and fore sights, and of the sums of the rises and falls, and shewing the work of casting to be correct. Thus are obtained a series of vertical heights to be set off perpendicularly to the datum line, through the upper extremities of which the sectional line must be drawn.

PRACTICAL LEVEL BOOK.

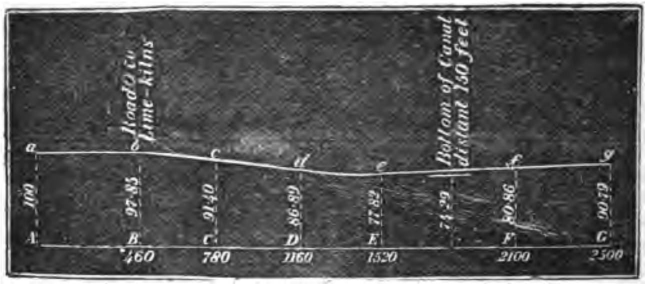
(Datum line 100 feet below the bench mark at *a*.)

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances	Remarks.
feet.	feet.	feet.	feet.	feet.	chains.	
				100·00D		
3·50	5·65		2·15	97·85	4·60	B M on road to lime kilns.
4·10	10·85		6·75	91·10	7·80	
5·04	9·25		4·21	86·89	11·60	
3·84	12·91		9·07	77·82	15·20	Bottom of canal, distant 2·80 chains.
4·12	7·65		3·53	74·29	……	
10·49	3·92	6·57		80·86	21·00	
12·96	3·03	9·93		90·79	27·00	to B. M. at *g*.
44·05	53·26	16·50	25·71	100·00		
	44·05		16·50			
	9·21	diff. =	9·21	= 9·21		diff. between last reduced level and datum.

In laying down the sectional line from the above columns of reduced levels and distances, the former are always taken from

a much larger scale than the latter, otherwise the undulations on the surface of the ground would, in many cases, be hardly perceptible.

Draw the horizontal line A G, setting off the distances A B, A C, &c., as in the column of distances, that is, A B = 4·60 chains, A C = 7·80, &c.; then draw A a = 100 feet, perpendicular to A G and parallel to A a draw B b, C c, &c., setting



off their heights 97·85, 91·10, &c., respectively from the column of reduced levels, and through the points a, b, c, &c., draw the required sectional line $a\ g$.

NOTE.—The above operations, though extremely simple, require great care, otherwise, in extensive works of this kind, errors creep in imperceptibly, to check which the agreement of the differences in the level-book is essential.

RUNNING A CHECK LEVEL.

(15.) We shall now give an example of a simple kind to lead the way to more complicated operations. When a section of a line of country has been completed, it is in most cases necessary to check its accuracy by repetition; but in doing this it is seldom requisite to level over precisely the same line of ground, unless there is cause to suspect its general correctness, but to follow the most convenient and nearest route, and at intervals to level to some known points on the exact line of section, which will give their differences of level: the points thus selected are generally what are called bench marks, being notches cut on gate posts, stumps of trees, mile or boundary stones, or any similarly immoveable objects, contiguous to the line of section, and at frequent intervals. These bench marks are made by the person who first takes the section, and are sometimes previously determined upon. When the section is complete, their relative heights with respect to the base or

datum line of the section becomes known; hence they may be considered as so many fixed points on the line, easily recognisable, from whence any portion of the work may be levelled over and over again.

From what has been said it is clear that in taking running or check levels, the use of the chain and compass attached to the level is not wanted, the distances and bearings having all been previously taken.

An example of this kind of operation is represented in Plate I., where both the ground plan and section are shewn. The strong black line on the plan is that of the section to be checked, and extends from a bench mark at the town A, in a winding direction, to another bench mark in a town B; this originally formed a portion of a more extensive survey. The route taken in proving the work is shewn on the plan by a dotted line, and was confined to the public roads, as being the most convenient route, especially as it crossed the line several times, by which a number of intermediate points could be checked. Previous to giving the particulars of this example, the method of performing the operation of levelling shall be explained more in detail than has yet been done.

The staff-holder must first place his staff perpendicularly on the bench mark from whence the levels are to commence; the surveyor must next set up his spirit level on the most convenient ground that presents itself, and so that he may have an uninterrupted view of the line he intends to level; the station chosen should not be more than four or five chains from the staff-holder, where, having fixed the legs of the level firmly in the ground, the surveyor must adjust his level for observation in the following order:—"first he must draw out the eye-piece of the telescope till he can see the cross wires perfectly well defined; then, directing it to the staff, he must turn the milled-headed screw, on the side of the telescope, till he can likewise distinguish, with the utmost possible clearness, the smallest graduations on the staff: that these two adjustments be very carefully and completely performed, is of more consequence than is generally supposed, for on them depends the existence or non-existence of parallax."

"The adjustment of the eye-piece to obtain distinct vision, when once properly made, is not likely to require alteration for the whole day, unless it be accidentally deranged; but that of obtaining distinct vision of the distant staff (together with the one we shall next describe) must be performed at every station as it varies with the distance of the staff.

"Having made the above adjustments perfect, bring the spirit bubble to the centre of its run, which position it must retain in every direction of the telescope; or, in other words, the bubble must indicate a true level during the time the telescope is turned completely round; this is accomplished by bringing the bubble successively over each pair of parallel-plate screws, and giving them motion by screwing up one while unscrewing the other to a like extent; but if the telescope is supplied with a cross level, as in that of Gravatt's, the two bubbles being at right angles to each other, they will at once shew which pair of screws require turning, in order to indicate the true level.

"The level being now adjusted for observation, it must be directed to the back staff, and with all possible exactness the foot and decimal fraction of a foot must be noted, with which the central part of the horizontal wire appears to be coincident, which enter in the proper column of the level book. This column should be headed 'Back Sights' as in the preceding example (Art. 14). As soon as it is registered, see that the bubble has not removed from its central position, and then repeat the observation to ensure that no mistake has been made: this should be invariably done to guard against error."

The telescope of the level must now be turned round to the fore staff, the object glass and bubble being again adjusted, if required, in the manner already described, and the reading of the staff observed and entered in the level book in the column headed "Fore Sights," verifying the observation as before by seeing that the bubble is still in its place, &c. It may be here proper to remark, that the surveyor's merely walking round the instrument, or accidentally striking one of its legs with his foot, will derange the level indicated by the bubble, especially where the ground is newly tilled or soft and spongy: therefore, the legs of the instrument should always be fixed firmly in the ground. To do away with this inconvenience, as far as possible, Gravatt's level (Art. 5.) has a reflector fixed on the top of the telescope, by which the observer can see both the staff and the reflected image of the bubble at the same time, and thus can make his observation at the instant he sees the bubble in its proper position. The foregoing description of the method of taking levels is general, and applies equally to every kind of levelling operations, we shall hereafter add such additional particulars as will require attending to, when taking levels for the formation of a section. The two first observations being com-

pleted, the surveyor must remove the instrument to the next station and set it up a second time, the person who held the back staff, removing it as far beyond the instrument as the instrument is beyond the staff, which has now become the back staff, and which must not be moved, except that its graduated side must be turned to the instrument. The observations may now be repeated in every respect the same as the two last described, and so on to the end of the work; excepting in cases where "long sights" are taken, in which the correction for curvature and refraction must be applied, as in the example (Art. 11.); but these cases seldom occur in practice. The explanations just given, are of a more detailed character than those given in (Art. 10.), as it must be borne in mind that this work is for the instruction of those who are unacquainted with the subject, and who ought not to be led into tedious details all at once.

The method of taking check levels being now explained, we shall next refer to the example, in which as already stated, "the levels were taken along the public road shewn by the dotted line (Plate I.,) as being the most convenient route between the town A and B, avoiding the necessity of passing through private property; the strong black line on the plan shews where the original section was taken; the section itself is shewn above the plan, and is drawn from two scales, the one giving horizontal measure, is the same as that of the plan, that is, one inch to the mile; and the vertical scale $\frac{3}{4}$ inch to the 100 feet. From this section it appears that the crown of the bridge at A is 14 feet above the datum line D E of the section, and that the bench mark (a stone by the road side,) at B is 111 feet above the same datum; therefore the difference of level between the two places is $111 - 14 = 97$ feet. Now by referring to our level book, of which we have subjoined a copy, we make the difference of level to be 96·8 feet, differing from the original section only 0·2 of a foot, or 2·4 inches, a quantity that may be disregarded; the inference to be drawn from such a coincidence in the two results is, that the whole of the section between the points in question is sufficiently correct.

LEVEL BOOK FOR RUNNING OR CHECK LEVELS.

Back Sights.	Fore Sights.	Remarks.
0·34	3·16	Back ⊙ on B.M.* on the bridge at A.
5·86	5·61	
4·19	4·24	Forward ⊙ at corner of road leading [to B.
5·44	1·20	
4·96	3·20	
4·73	1·32	At crossing of line.
6·10	2·00	
5·33	3·96	
5·91	1·83	
3·70	0·90	
6·02	1·21	Staff placed on post notched for B.M.
1·21	4·00	At crossing of line.
3·53	6·07	
3·96	5·34	
3·94	4·81	
3·98	6·08	
4·08	4·94	Upon line.
3·90	2·96	
4·84	2·42	
1·54	5·12	
4·69	4·97	
5·04	1·60	
2·24	3·86	Upon line.
7·25	1·89	
4·03	1·30	
9·54	0·19	
6 70	1·70	
9·40	4·06	
4·44	1·38	
140·89	91·32	Sums
91·32		
49·57		Difference.

* B. M. signifies bench mark.

Back Sights.	Fore Sights.	Remarks.
49·57		Brought forward.
11·00	0·46	
5·98	1·30	
11·12	1·78	
9·84	2·20	
0·18	0·32	Upon line.
4·72	0·10	
8·89	0·77	
10·02	0·92	
10·00	1·03	
8·58	1·19	
9·53	1·18	
9·90	1·68	
9·04	0·35	
10·00	8·52	
3·00	11·55	
3·68	0·88	
7·21	8·75	
1·99	10·48	
0·65	10·00	
4·48	10 44	
7 47	10·30	
1·55	11·70	
2·45	9·88	
3·78	1·04	
1·64	1·65	Forward ⊙ on B. M. called B.
205·27 108·47	108·47	Sums.
96·80	Diff. =	Difference of levels between A and B.

The difference of the sums of the back and fore sights being nearly 97 feet, proves that the bench mark at B is higher by that distance than the bench mark at A, as shewn by the section.

LEVELS FOR THE FORMATION OF A SECTION.

(16.) The next most simple case that can occur, is to take the levels of a line of country, where the ground plan is already made, and the line of section determined upon and marked out on the plan. Here, in addition to what is required in running or check levels, the distances to the several stations of the levelling staves from the starting point must be measured. Plate II. represents an example of this kind of work, the survey of the land having been completed, and the plan of the fields, &c., drawn: the strong black line A B was the direction determined upon, as the most suitable for an intended line of railroad, and the section was accordingly taken:—bench marks had been previously agreed upon at the extremities A and B, from whence other surveyors could take up the levels and carry them onwards in both directions.

Two additional assistants are required in this case, to measure the distances of the stave stations along the lines while the operation of levelling goes on, which is the same in every respect as that already described for check levels, excepting that, in this case, the operation is conducted upon the strong black line A B, on the surface plan, a copy of which must be in the surveyor's possession to direct him, and the distances of the several stave stations must be noted in the level book, in the column marked "Distances."

The following is the level book of the example given in Plate II., shewing the manner of keeping it, and also the method of reducing the levels to obtain the actual heights of each station above the datum line E F, which is placed 100 feet below the starting point A, for convenience of drawing the section. The whole operation being similar to that already given at Art. (14), excepting that here and in the preceding example we give the particular manner of performing the several parts of the field work, in order that it may be clearly understood by those who are unacquainted with the subject.

THE LEVEL BOOK FOR PLOTTING THE SECTION.

(Datum 100 feet below the station A.)

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances.	Remarks.
feet.	feet.	feet.	feet.	feet.	links.	
				100·00 D		
13·71	7·88	5·83		105·13	519	B. M. side of road.
9·40	16·30	t	6·90	98·93	1315	
3·87	11·71		7·84	91·09	1542	
2·63	12·41		9·78	81·31	1850	
14·62	0·95	13·67		94·98	2358	
17·00	1·45	15·55		110·53	2698	
10·66	15·40		4·74	105·79	3357	
2·87	17·00		14·13	91·66	3758	
3·40	10·32		6·92	84·75	3976	
5·73	2·24	3·49		88·23	5077	
16·54	0·85	15·69		103·92	5904	
16·08	0·89	15·19		119·11	6124	
14·56	0·73	13·83		132·94	6437	
10·36	14·06		3·70	129·24	7467	
9·84	1·36	8·48		137·72	8369	
9·80	7·00	2·80		140·52	9303	
2·30	10·96		8·66	131·86	——	Centre of road at 215
10·96	14·46		3·50	128·36	9679	[links.
2·08	15·05		12·97	115·39	9936	
1·75	16·58		14 83	100·56	10164	
1·84	17·10		15·26	85·30	10576	
0·00	7·43		7·43	77·87	11423	Forward ⊙ at corner
5·38	3·50	1·88		79·75	13066	[of wood.
8·50	4·50	4·00		83·75	14954	
5·30	1·36	3·94		87·69	15650	
10·20	9·40	0·80		88·49	17345	
6·86	0·40	6·46		94·95	19135	
11·00	3·96	7·04		101·99	19359	
11·80	3·53	8·27		110·26	19631	
10·53	2·68	7·85		118·11	19841	Forward ⊙ at end of
8·82	1·98	6·84		124 95	20561	[wood.
8·76	2·20	6·56		131·51	21671	
14·00	14·50		0·50	131·01	——	Road at 450 links.
14·50	4·32	10·18		141·19	22710	
9·14	1·00	8·14		149·33	23221	
304·19	254·86	166·49	117·16	100·00		
254·86		117·16				
49·33	—	49·33	—	49·33	Difference between Datum and last Reduced level, or height of B above A.	

The several differences of the sums of the back and fore sights, of the sums of the rises and falls, and of the last reduced level and the datum, exactly agreeing, proves the accu-

racy of the arithmetical operation in the preceding level book, all these differences being 49·33 feet, which is the height of B above A, in the section Plate II.

It is advisable for the surveyor to reduce the levels in the field as he proceeds, as it will occupy very little time and can be easily done while the staffman is taking a new position. The surveyor will thus be enabled to detect with the eye if he is committing any glaring error; for instance, inserting a number in the column of rises, when it ought to be in that of falls, the surface of the ground at once reminding him that he is going downward instead of ascending.

It is seldom the case in practice that the instrument can be placed precisely equi-distant from the back and fore staves, on account of the inequalities of the ground, ponds, &c.; it would appear, therefore, to be necessary, to make our results perfectly correct, to apply to each observation the correction for curvature and refraction as explained in Art. (7): this, we believe, is seldom done, unless in particular cases where the utmost possible accuracy is required, on account of the smallness of such correction, as may be seen by referring to the table at the end of the book, where this correction for 11 chains is shewn to be no more than $\frac{1}{100}$ part of a foot; and as the difference in the distances between the instrument and the fore and back staves can in no case equal that sum, it is evident that such correction may be safely disregarded in practice. Besides it is not necessary to have the level placed directly between the staves while making observations, as it is frequently inconvenient to do so, for reasons just given, nor does a deviation from a line of the staves, in this respect, in the least affect the accuracy of the result.

The distances in the sixth column of the level book are assumed to be horizontal distances, and in measuring them, care should be taken that they are as nearly such as possible, or they must be afterwards reduced thereto, otherwise the section will be longer than it ought to be. For the purpose of assisting the surveyor in making the necessary reduction from the hypothenusal to the horizontal measure, when laying down the section, we annex the following table, shewing the reduction to be made on each chain's length for the following quantities of rise, as shewn by the reading of the staves.

Rise in feet for one chain.	Reduction on one chain in links and decimals.	Rise in feet for one chain.	Reduction on one chain in links and decimals.
1	0·01	11	1·40
2	0·04	12	1·66
3	0·11	13	1·92
4	0·19	14	2·24
5	0·27	15	2·61
6	0·44	16	2·99
7	0·56	17	3·39
8	0·74	18	3·76
9	0·94	19	4·23
10	1·16	20	4·64

TO DRAW THE SECTION.

The levels being reduced, as well as the distances corrected, where required, the surface line may be represented in the form of a section, as shewn in the upper part of Plate II. The vertical and horizontal scales of a section are seldom the same, for the reason assigned in Art. (14.), which produces a caricatured representation of the surface of the section; the vertical scale being so much greater than the horizontal, shews the depths of the cuttings and embankments, required in the execution of railways, canals, &c., with greater clearness than if both scales were alike. The plans and sections of projected work, deposited in the Private Bill Office, to obtain the sanction of the legislature, are mostly drawn to a scale of 13½ inches to one mile horizontal and one hundred feet to one inch vertical.

To make the section expeditiously, first draw the horizontal line E F, as the datum to which the levels were reduced, take any point E as the starting point, place the feathered edge of the horizontal scale against E F and prick off the several distances in the column headed "Distances," that is, 5·19, 13·15, 15·42 chains, &c., then draw all the perpendiculars by means of a parallel ruler, or by a T square if the paper is properly fixed on a drawing table; and lastly from the vertical scale prick off all the reduced levels or vertical heights corresponding to the several distances, connect the points and the section will be made, after which, write, in vertical lines, the several remarks, as crossing of roads, rivers, &c., that appear in the last column of the level book. The horizontal line A D, called the "datum from bench mark at A," shews the variations of the surface of

the section above and below the point A, from which it was originally plotted by Mr. Simms, his level book being adapted thereto.

CROSS LEVELS.

When a road or river crosses the line of a railway, or canal, cross levels are principally taken to shew the nature of the surface of the ground, both with respect to improving the main line, if possible, and to shew the approaches of cross roads to the viaducts, when required, and the depth and length of cutting, or the height and length of embankment that will be required, where the main line is either crossed on the level, or over or under by viaducts. The heights of the cross section is usually taken at every 1 or 2 chains length to the distance of 8 or 10 chains on each side of the main line; but if the cross road have a regular slope, it will be sufficient to shew the inclination of the slope, which may be done by a single setting up of the level and staves. The following notes will shew the method of taking a cross section, recollecting always to take the levels from the right to the left of the forward direction of the main line, lest the section, through mistake, should be plotted in the wrong direction.

CROSS LEVELS ON ROAD AT 92·15 CHAINS.

(Section, Plate II.)

Back Sights.	Fore Sights.	Distances.	Remarks.
feet.	feet.	chains.	
1·84	0·16	2·00	
1·02	8·70	3·00	
2·83	7·91	4·00	
5·20	10·63	6·00	
4·13	8·71	7·92	Line crosses reduced level 131·86 feet.
4·04	8·02	10·00	
2·92	7·92	12·00	
3·16	6·04	14·00	
2·17	6·87	15·00	
2·13	7·00	16·80	

The cross section may be plotted on the same scale as the main section; but some engineers adopt a larger one. From what has been already shewn, the student can have no difficulty in plotting from the preceding notes.

PARLIAMENTARY PLAN AND SECTION.

These differ little from what is shewn on Plate II, only in the former the fields, &c., are numbered with an accompanying book of reference to proprietors' names, &c.; and the cross sections are added in the latter, and the gradients put on, as described at page 153. See Working Section, Plate III., and the Standing Orders of the two Houses of Parliament, in No. 62 of Weale's Series.

WORKING SECTION.

When the works of a railway, canal, &c., have to be carried into execution, the section must be more minutely taken than would be required in the preceding cases; it is then called a working section. The following are the field notes for such a section. Plate III. shews the section of the ground and railway at the extreme end of the line, where the distances terminate at 1103·77 chains, or 13¾ miles and 3·77 chains. The student will be able to plot the section from the following level book and the accompanying directions. In taking levels for this section, the back and fore sights are not very far distant from each other, and the surveyor will frequently be able to make a number of observations at each setting up of the instrument both in the back and forward directions, the back staff being repeatedly removed nearer the instrument, where the ground varies, and the fore staff also repeatedly removed farther from it: in this manner from seven to ten observations may be obtained at one setting up of the instrument, if required. In the following level book, it will be seen, that it seldom occurred that only one back and one fore sight was obtained at each setting up of the instrument: at the first setting up four forward sights were observed; thus the first back sight was 4·47 and the corresponding fore sights 4·53, 9·22, 5·07, &c. Here the first fall is obtained in the usual manner, that is, by taking the difference between 4·53 and 4·47; the next fall is obtained by taking the difference between the first and second fore sights, that is, between 9·22 and 4·53, this difference is 4.69, which is a fall, because the latter fore sights is the greater; the third difference between the second and third fore sights is a rise, because the latter fore sight in this case, is the less: and so on till we come to the next back sight 6·36, when its corresponding fore sight 1·87 is taken from it, as previously shewn, and the difference placed in the column of rises. The column of reduced levels is obtained as in the preceding examples.

LEVEL BOOK.—WORKING SECTION.

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances.	Remarks.
feet.	feet.	feet.	feet.	feet.	chains.	Height above Trinity high water-mark, at London Bridge.
				270·72		
4·47	4·53		0·06	270·66	1033·00	
	9·22		4·69	265·97	1034·00	
	5·07	4·15		270·12	1035·00	
	0·24	4·83		274·95	1036·00	
6·36	1·87	4·49		279·44	1037·00	
6·14	1·47	4·67		284·11	1038·00	Side of falling post of field gate in occupation road.
6·62	2·10	4·52		288·63	1039·00	
	2·24		0·14	288·49	1039·16	
10·42	9·47	0·95		289·44		Lower hinge of gate.
	13·22		3·75	285·69	1039·44	Centre of occupation road
	13·15	0·07		285·76	1039·56	Edge of road.
	8·75	4·40		290·16	1039·66	Top of bank.
	4·48	4·27		294·43	1039·76	Ditto.
	4·32	0·16		294·59	1040·00	
2·44	8·84		6·40	288·19		B. M. south side of line.
	2·83	6·01		294·20	1041·00	
0·74	2·18		1·44	292·76	1042·00	
	5·35		3·17	289·59	1043·00	
6·77	7·28		0·51	289·08	1044·00	
	7·25	0·03		289·11	1044·90	Edge of ditch.
	8·36		1·11	288·00	1044·92	Bottom of ditch.
	3·57	4·79		292·79	1045·00	Stump, top of bank.
3·37	2·75	0·62		293·41	1046·00	
	1·43	1·32		294·73	1047·00	
1·10	2·25		1·15	293·58	1048·00	
	8·88		6·63	286·95	1049·00	Enter alder plantation.
5·65	9·53		3·88	283·07	1049·20	
	11·50		1·97	281·10	1050·00	
5·85	5·52	0·33		281·43	1050·21	
	12·01		6·49	274·94	1051·00	
	12·87		0·86	274·08	1051·48	
	10·77	2·10		276·18	1051·90	Foot of bank, which rises perpendicularly 2 feet.
	8·59	2·18		278·36	1052·00	
	1·40	7·19		285·55	1053·00	
8·22	4·42	3·80		289·35	1054·00	
	2·97	1·45		290·80	1055·00	
	3·39		0·42	290·38	1056·00	
	5·51		2·12	288·26	1057·00	
	7·67		2·16	286·10	1058·00	
5·41	6·68		1·27	284·83	1058·27	Edge of ditch.
	8·56		1·88	282·95	1058·32	Bottom of ditch.
	6·08	2·48		285·43	1058·37	Top of bank.
	12·38		6·30	279·13	1058·54	Foot of bank.
	16·72		4·34	274·79		
2·04	2·66		0·62	274·17	1059·00	
	6.48		3·82	270·35	1059·40	Edge of ditch.

LEVEL BOOK OF WORKING SECTION (*continued*).

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances.	Remarks.
feet.	feet.	feet.	feet.	feet.	chains.	
				270·35		Brought forward.
	8·86		2·38	267·97	1059·44	Bottom of ditch.
	6·00	2·86		270·83	1059·52	Top of bank.
	7·58		1·58	269·25	1059·60	Foot of bank.
	10·74		3·16	266·09	1060·00	
3·33	8·24		4·91	261·18	1060·95	Top of bank.
	9·15		0·91	260·27	1061·00	Stump side of bank.
	13·34		4·19	256·08	1061·05	Bottom of ditch.
	11·65	1·69		257·77	1061·10	Edge of ditch.
	12·80		1·15	256·62	1062·00	
3·62	4·13		0·51	256·11	1063·00	
	3·38	0·75		256·86	1063·49	Foot of bank.
	0·50	2·88		259·74	1063·59	Top of band.
	4·35		3·85	255·89	1063·68	Bottom of drain.
	4·08	0·27		256·16	1063·86	Centre of parish road.
	4·31		0·23	255·93	1064·05	Foot of bank.
	0·57	3·74		259·67	1064·15	Top of bank.
	3·02		2·45	257·22		
0·99	1·40		0·41	256·81	1064·30	Foot of bank.
	2·83		1·43	255·38	1065·00	
	4·41		1·58	253·80	1066·00	
	4·48		0·07	253·73	1067·00	
7·80	7·34	0·46		254·19	1068·00	(Crosses foot path at [1068·31.)
	4·41	2·93		257·12	1069·00	
	0·74	3·67		260·79	1070·00	
10·63	6·43	4·20		264·99	1071·00	
	1·37	5·06		270·05	1072·00	
10·76	4·97	5·79		275·84	1073·00	
	1·12	3·85		279·69	1074·00	
5·42	5·47		0·05	279·64	1075·00	
	4·56	0·91		280·55	1076·00	
	5·00		0·44	280·11	1076·37	Edge of ditch.
	5·56		0·56	279·55	1076·40	Bottom of ditch.
	2·40	3·16		282·71	1076·47	Top of bank.
	2·96		0·56	282·15	1076·54	Foot of bank.
	1·38	1·58		283·73	1077·00	
9·45	0·61	8·84		292·57	1078·00	
8·44	3·33	5·11		297·68	1078·53	Enter plantation.
	0·42	2·91		300·59	1078·57	B. M. on timber stump
12·78	14·28		1·50	299·09	1078·82	
	10·01	4·27		303·36	1079·00	
	1·26	8·75		312·11	1079·47	
14·49	4·07	10·42		322·53	1080·00	
	2·86	1·21		323·74	1080·08	
	3·35		0·49	323·25	1080·24	
	0·37	2·98		326·23	1080·47	
16·35	1·02	15·33		341·56	1080·98	Top of bank, edge of [plantation.

LEVEL BOOK OF WORKING SECTION (*continued*).

Back Sights.	Fore Sights.	Rise.	Fall.	Reduced Levels.	Distances.	Remarks.
feet.	feet.	feet.	feet.	feet.	chains.	
				341·56		Brought forward
	1·32		0·30	341·26	1081·00	
8·66	5·92	2·74		344·00	1082·00	
	5·14	0·78		344·78	1083·00	
	8·43		3·29	341·49	1084·00	
1·05	4·50		3·45	338·04	1085·00	
	4·94		0·44	337·60	1085·20	
	6·83		1·89	335·71	1085·30	Edge of bank.
	12·54		5·71	330·00	1085·40	Foot of bank.
	16·82		4·28	325·72	1086·00	
1·11	9·04		7·93	317·79	1087·00	
1·18	9·09		7·91	309·88	1088·00	
1·57	9·70		8·13	301·75	1089·00	
1·28	9·58		8·30	293·45	1090·00	
1·44	9·41		7·97	285·48	1091·00	
1·34	9·14		7·80	277·68	1092·00	
1·15	8·12		6·97	270·71	1093·00	
3·04	4·43		1·39	269·32	1093·86	Edge of ditch.
	6·22		1·79	267·53	1093·90	Botton of ditch.
	5·16	1·06		268·59	1094·00	Stump, top of bank.
	11·10		5·94	262·65	1094·08	Foot of bank.
	9·29	1·81		264·46	1095·00	
8·87	5·77	3·10		267·56	1096·00	At post and rail fence.
4·63	3·46	1·17		268·73	1097·00	Edge of slope.
	7·06		3·60	265·13	1098·00	
1·96	4·60		2·64	262·49	1099·00	Foot ot slope.
	4·60			262·49	1100·00	
	3·67	0·93		263·42	1101·49	
4·58	5·03		0·45	262·97	1103·77	Stump, end of curve
	4·40	0·63		263·60		
	4·58		0·18	263·42		B. M. foot of post.
	1·05	3·53		266·95		Top of said post.

This method of keeping the level book is adopted by many surveyors; the accuracy of the castings is proved by taking the difference of the sum of the back sights and the sum of the last fore sights, leaving out all the intermediate sights, and the difference of the last reduced level, and the same brought forward, and when these differences agree, the castings are correct. Some put the intermediate sights in a separate column, in this case a somewhat different method of casting is adopted:—I have given this method at page 357 of my system of levelling in the ninth edition of Nesbitt's Surveying.

PLOTTING THE WORKING SECTION.

(See Plate III.)

Having drawn the datum line, prick off every chain and number them beneath the datum line to prevent mistakes. Next prick off the distances in the column of distances, and erect the perpendiculars, and just above the datum line, the height at each chain's length should be inserted, from the column of reduced levels; some also insert the heights of the intermediate stations, that the section may be plotted without the fear of errors. The horizontal scale in the example is 1 inch to 5 chains, and the vertical scale 1 inch to 25 feet; these scales are frequently adopted in parliamentary plans. Having laid off all the heights on the perpendiculars, the undulating line forming the surface of the section may be readily drawn, as shewn in the plate referred to, and the description of objects worthy of notice must be added. The section is then prepared for putting on the gradients, &c.

THE METHOD OF LAYING OUT GRADIENTS.

Gradients are evenly ascending or descending portions of a railway; the ascent or descent being always less than 1 foot, reckoned vertically, to 100 feet on the level, or estimated horizontally; thus to afford the means of rapid locomotive traction. Some gradients, however, are level, and such gradients are preferable to any other; but the undulations of the earth's surface prevent their adoption to any great extent, in by far the greater portion of railways.

The extreme left hand point of the section M, Plate III., being the point of junction with another railway, is the place from whence the gradients are to be laid out: the railway is represented by two parallel lines, the upper one being the surface of the rails, and the lower one the bottom of the ballasting or formation level, being $2\frac{1}{4}$ feet below the surface of the rails. For a short distance M N the gradient is level; the gradient N O then rises at the rate of 20 feet per mile, or one foot in 264, for the twofold object of diminishing the great cutting and getting sufficiently high over the road at stake 1064 to allow headway for public carriages to pass under the railway. From this point the gradient O P falls with the same rate of inclination for a considerable distance, the object being to get as low down as convenient further to the eastward where there would be a considerable embankment required; thus reducing the extent of both the cuttings and embankments. Each change of gradient is marked by a strong vertical line from the datum

line to the point of change, and the height written thereon The quantities of earthwork to form the cuttings and embankments with different slopes should be written on them, as shewn in the example (the method of finding these quantities shall be hereafter shewn); also above the line of figures denoting the height of the surface above the datum, should be placed the depth of the cutting from the surface to formation level at the same point, or the height of the enbankment, as the case may be: these heights and depths are those from which the calculations of the quantities of earthwork are to be made, and therefore must be strictly correct: these heights and depths are frequently taken by measuring them carefully with the vertical scale of the section, but they may be more correctly obtained by calculation in the following manner. Let it be required to find the depth of the cutting at stake No. 1083, when the height of the surface above the datum is 344·78 feet; at stake No. 1064 the height of formation level above datum is 269·20, from which point the gradient descends at the rate of 20 feet per mile, or 0·25 feet per chain, towards No. 1083; the distance from 1064 to 1083 is 19 chains, which multiplied by 0·25 gives 4·75 for the fall of the railway in the space between the two points; hence the height of the railway above datum at No. 1083 is $269{\cdot}20 - 4{\cdot}75 = 264{\cdot}45$; this sum subtracted from the whole height of the surface gives $344{\cdot}78 - 264{\cdot}45 = 80{\cdot}33$ for the depth of the cutting at that point, and so on for all the remaining numbers. In the same manner the heights of bridges and viaducts may be found, either below or above the railway, recollecting to subtract the reduced level from the height of the gradient in the former case.

It will here be proper to inform the student that the common, and, perhaps only practical method of laying out the gradients of railways, is by applying one end of an extended silken thread to the section at the commencement of the railway, the other end being so applied that the thread may cut the profile of the earth's surface, so as to leave equal portions of space both above and below the thread, judging by the eye, that the cuttings from the parts above the thread may furnish sufficient materials to fill up the spaces or parts below the thread to form the embankments. If the position of the first gradient, though in itself favourable, should cause the next gradient to be less favourable, with regard to the extent of cuttings and enbankments, the position of the first gradient must be altered to suit the next following gradient, till it be found that the compound result of the cuttings and embankments on the two gradients,

or on the successive gradients, as now altered, shall have less of cuttings and enbankments than in the preceding case. In this way it is advisable to change the positions of the gradients till the *minimum* of cuttings and embankments seems evidently to be attained, due regard being had to the limit in the ascent or descent of the gradients, which, as before observed, ought not to exceed 1 in 100, also keeping in view, at the same time, the proper height for bridges to cross rivers, &c., in the meantime the difficulty of making the excavations being supposed throughout the length of the line.

Great diversity of opinion prevails among engineers as to the propriety of making the gradients subservient to the economical construction of railways; hence we find, in many cases, a wide departure from the rule, as applied in the earlier days of railways. Some of our most eminent engineers of the present time are in the habit of laying out what are termed "severe gradients," so that inclinations of 1 in 80, and even as far as 1 in 60 are frequently found in the sections of a great number of the new and branch lines, which of course is a question of expediency, between present saving in construction and the future cost of working the lines.

In laying out the gradients, it is desirable to affect as little as possible the existing levels of public roads; which, if practicable, should be crossed either on the level or 20 feet above or below them; if impracticable, the road must be raised or sunk to meet the level of the rails, as the case may require; and sunk to gain the depth of 20 feet, in case of its passing under the railway, the inclination of its approaches to the railway being made 1 in 30 in turnpike, and 1 in 20 in other roads; also, if practicable, all stations should be placed at the top of two gradients descending both ways; as such gradients both serve to check the speed of a train, when approaching such a point, and assist it to regain its speed when leaving.

The height of the gradient over or depth under the surface of any turnpike or public carriage road, existing railway, river, or canal, must be marked in figures on the section at each crossing thereof, as well as the height and span of the arch, or arches, forming the viaducts, by which the crossing is intended to be effected.

RULE FOR FINDING THE RATE OF INCLINATION OF A GRADIENT.

Divide the horizontal length of the gradient in feet, by the difference of the heights of the gradient at its extremities, above the datum line, and the quotient is the horizontal to a rise or

fall of one foot, which is called the rate of inclination of the gradient.

EXAMPLE.

In Plate III. the second gradient at its commencement, is 261·35 feet, and at its termination it is 269·20 feet above the datum line; thus giving a rise of 269·20 — 261·35 = 7·85 feet, the horizontal length of the gradient is 1095·50 — 1064 = 31·50 chains = 2079 feet, whence 2079 ÷ 7·85 = 264, (fractions being omitted) or a rise of 1 in 264, or of 20 feet per mile, as shewn on the section in the plate referred to.

TUNNELS.

When the excavations reach the depth of 60 feet, and continue at that, or a greater, depth for a considerable distance, the most economical method of proceeding with the work is to make a subterraneous passage called a tunnel, through these deep parts; for it would be next to impossible in many such cases to cut the ground open to the surface. Tunnels, on railways of the narrow gauge, are usually cut to the width and depth of 25 or 30 feet, and on railways of the broad gauge they are proportionably larger. The width and depth of the tunnel are less, if the material to be cut be hard rock. A tunnel A B C D, on the gradient N O is shewn in Plate III.; its length A B is 7 chains or 154 yards, and its height A C 25 feet. All tunnels must slope to one or both of their extremities for the purpose of drainage; and it will be seen that the tunnel, here referred to, slopes to the end A. In laying out the gradients the diminished quantity of cuttings, where there are tunnels, must be taken into account. This subject shall be resumed hereafter.

LEVELLING WITH THE THEODOLITE.

The use of the theodolite is sometimes necessary in levelling operations, especially when these operations are required to be conducted over very high and rapidly rising ground, or over steep and almost perpendicular rocks, where the ordinary levelling instrument cannot be fixed. Select a convenient place to fix the theodolite, where the general inclination of the surface of the country changes, without regarding its minor inequalities; then set the instrument level by means of the parallel plate screws, and direct an assistant to go forward with a staff, having a vane or flag fixed to it, of the same height from the ground as the centre of the axis of the telescope of the theodolite. Having gone to the station required, the assistant must hold the vane staff upright, while the observer measures the vertical angle, which an imaginary line, connect-

ing the instrument and staff, makes with the horizon. The instrument and staff should then change places, or, to save time, another staff should take the place of the instrument, the instrument being removed to the former staff, and from thence the angle should again be taken to the second staff; the *mean* of which two angles may be considered the correct angle. This precaution is necessary on account of the variableness of the refraction, and more especially so where the points of observation are at a great distance, and one much higher than the other. The distance on the slope must be measured in the mean time, which, with the mean angle, constitute the hypothenuse and angle at the base of a right angled triangle, in which the base is the horizontal distance between the two stations, and the perpendicular their difference of level, both of which may be readily found by trigonometry, or by laying down the triangle and measuring the parts in question.

In this manner, by considering the surface of every principal undulation as the hypothenuse of a right angled triangle, the operation of levelling may be carried on with great rapidity; but it must be remarked, without pretensions to strict accuracy, —in fact, in this particular the use of the spirit level can never be superseded.

LEVELLING BY THE BAROMETER.

The method of finding the difference of levels for railway purposes by the barometer, though frequently recommended, will be found to fail in point of accuracy, on account of the sudden changes in the pressure of the atmosphere, on which depend the indications of this instrument, since 90 feet elevation correspond to only one-tenth of an inch of the mercurial column, which difference has frequently been noticed at the same place, in a very short space of time, the weather at the same time being apparently settled. This method, therefore, can never be relied upon further than as a rough approximation.

CHAPTER II.

THE METHOD OF LAYING OUT RAILWAY CURVES, TURNOUTS, CROSSINGS, ETC.

ON RAILWAY CURVES IN GENERAL.

The natural unevenness of the earth's surface renders the use of curves in railways absolutely necessary, in order that the nearest practicable level may be secured, by avoiding mountains, crags, and other minor elevations, by winding round their bases by means of curves; which are also equally required to avoid various other natural and artificial obstructions, as rivers, lakes, sea-coasts, swamps, &c.; also towns, parks, pleasure-grounds, &c. Thus a great saving is effected in the cost of construction, and in the severance of valuable property, which would be otherwise required. Besides, winding railways are frequently required, in order that they may embrace in their routes important towns, harbours, mineral districts, &c., or make junctions with other railways.

In railway practice the curve adopted is always an arc of a circle; and sometimes two, three, or more consecutive arcs of circles of different radii, having a common tangent or tangents at their point or points of junction, as in the *compound curve.* Frequently the railway curve is composed of two or more circular arcs, having their convexities turned in different directions, with a common tangent or tangents at their point or points of junction; a curve thus composed is called the *serpentine or* S *curve.* The straight portions of the railway are always first laid out, and are, in all cases, tangents to the curves at their *termini.*

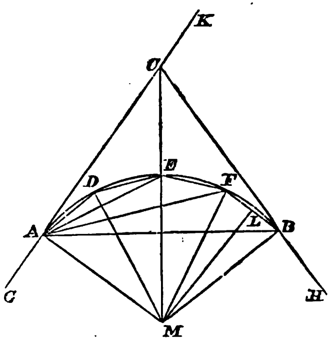
It has been found in practice that at least four different methods of laying out railway curves on the ground are requisite, on account of obstructions on the ground, such as buildings, cliffs, woods, rivers, &c., situated either on the convex or concave side of the curve, or on both sides, or on the curve itself; also on account of pits, bogs, swamps, &c., which either wholly or partly prevent the use of the surveying chain.

Railway curve-rulers are a series of arcs of circles of various radii, usually from 3 to 60 inches, and are used for projecting railway curves on parliamentary maps, &c.; and to determine the radii of curves already projected. The radius of

each curve-ruler is marked upon it in inches; and when a curve-ruler is applied to a railway map, the scale of which is 5 chains to an inch, the radius must be multiplied by 5 to obtain the true radius of the curve: thus, if the radius of the curve-ruler be 16 inches, then $16 \times 5 = 80$ chains = one mile, which is the radius of the curve; and so on for maps of other scales.

The limit of the radii of railway curves.—By the Standing Orders of Parliament, a *minimum* limit of one mile, or 80 chains, was formerly assigned to the radii of railway curves; because, in curves of less radii, a railway-train of great velocity has a tendency to run off the line on the convex side of the curve. This limitation is now very frequently dispensed with, by giving a proper super-elevation to the exterior rail of the curve to counterbalance the centrifugal force. (See formula for this purpose at p. 210.)

The following propositions relating to the circle are derived from geometry, and will be found useful in their application to railway curves.



1. A tangent to a circle is perpendicular to the radius drawn to the tangent point. Thus the tangent A C is perpendicular to the radius A M.

2. Two tangents drawn to a circle from any point are equal, and if a chord be drawn between the two tangents' points, the angles formed by the tangents with the chord are equal. Thus A C = B C, and the angle B A C = A B C.

3. An acute angle between a tangent and a chord is equal to half the central angle subtended by the same chord. Thus C A B = ½ A M B or A M C.

4. An acute angle having its vertex in the circumference of a circle, and subtended by a chord, is equal to half the central angle subtended by the same chord. Thus D A E = ½ D M E.

5. Equal chords subtend equal angles at the centre of a circle, and also at the circumference, if the angles are contained in similar segments. The chords A D, D E, E F,

and F B being equal, the angles A M D, D M E, E M F, and F M B are equal.

6. The angle K C B is called the angle of intersection of the tangents, and is equal to the central angle A M B subtended by the chord A B, which joins the tangent points A and B.

Problem I.

The positions of two straight or tangential portions of a railway, G A and H B, being given, to determine the radius of the curve that joins them. (See figure, p. 159.)

This problem seems to admit of an indefinite number of solutions, but when local obstructions, such as rivers, roads, hills, buildings, &c., are considered, it is often limited to one solution. Besides, by the Standing Orders of Parliament, it is required that "a memorandum of the radius of every curve not exceeding one mile in length shall be noted on the parliamentary plan in furlongs and chains;" and even in the case of curves having radii of one mile and upwards, no engineer in laying out a

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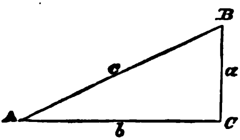
Concise Trigonometrical Formulæ.

Let A B C be a right triangle. If the angles be denoted by A, B, and C, and the sides by a, b, and c, as in the figure, we shall have these six formulæ:—



1. $\sin. A = \frac{a}{c}$ 4. $\text{cosec.} A = \frac{c}{a}$

2. $\cos. A = \frac{b}{c}$ 5. $\sec. A = \frac{c}{b}$

3. $\tan. A = \frac{a}{b}$ 6. $\cot. A = \frac{b}{a}$

Solution of Right Triangles.

	Given	Sought.	Formulæ.
7	a, c	A, B, b	$\sin. A = \frac{a}{c}, \cos. B = \frac{a}{c}, b = \sqrt{(c+a).(c-a)}$, or $\sqrt{c^2 - a^2}$
8	a, b	A, B, c	$\tan. A = \frac{a}{b}, \cot. B = \frac{a}{b}, c = \sqrt{a^2 + b^2}$
9	A, a	B, b, c	$B = 90° - A, b = a \cot. A, c = \frac{a}{\sin. A}$
10	A, b	B, a, c	$B = 90° - A, a = b \tan. A, c = \frac{b}{\cos. A}$
11	A, c	B, a, b	$B = 90° - A, a = c \sin. A, b = c \cos. A$

railway is justified, under any circumstances, in passing beyond the limits of deviation which are limited by the Standing Orders of Parliament to 100 yards on each side of the centre line as shown on the parliamentary plan.

Solution.—Let G A and H B be produced until they meet in C. Extend G A to K. Measure the angle K C B, which is equal to the central angle A M B, subtended by the curve; half this angle is equal to the angle A M C, which, subtracted from 90°, gives the angles A C M. The point A being the tangent point from which the curve is proposed to commence, measure the line A C. We have then given all the angles of the triangle A C M, and the side A C, by which the radius A M can be readily found by No. 6 formula, p. 160. By this formula $\frac{A\ M}{A\ C}$ = cot. A M C, or $\frac{R}{T}$ = cot. A M C; therefore Rad. or R = T cot. A M C.

EXAMPLE.

Given A M C = 22° 30′, and T = 950, to find R, or radius.

T = 950	log.	2·977724
A M C = 22° 30′	cot.	0·382776
R = 2293·5		3·360500

Method of laying out a curve by tangential angles.—The method generally adopted at present for laying out curves on

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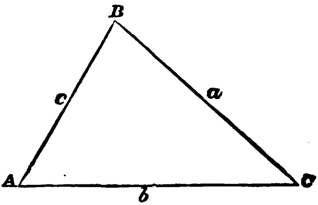
SOLUTION OF OBLIQUE TRIANGLES.



	Given.	Sought.	Formulæ.
12	A, B, *a*	*b*	$b = \frac{a \sin. B}{\sin. A}$
13	A, *a*, *b*	B	$\sin. B = \frac{b \sin. A}{a}$
14	*a*, *b*, C	A—B	$\tan. \frac{1}{2}(A - B) = \frac{(a - b) \tan. \frac{1}{2}(A + B)}{a + b}$

the ground is by means of tangential angles, or, as they are sometimes called, deflection angles.

Problem II.

Given the radius A M = R, to find the tangential angle C A D for a chord of one chain. (See figure, p. 159.)

Solution.—Draw M L perpendicular to B F. Then the angle B M L = ½ B M F = ½ A M D = C A D, the tangential angle, and B L = ½ B F = 50 links. But in the right triangle M B L we have (formula 1, p. 160) sin. B M L $= \frac{\text{B L}}{\text{B M}}$.

$$\therefore \text{sin. C A D} = \frac{50}{\text{R}}.$$

EXAMPLE.

Given R 40·00 to find C A D.

50	log.	1·698970
40·00		3·602060
C A D = 42′ 58″		8·096910

Second method of ascertaining the tangential angle.—Divide the constant 1718·9 by the radius in chains, which will give the angle in minutes.

In the present case the radius is 40 chains. Then $\frac{1718 \cdot 9}{40} = 42 \cdot 972$ minutes, or 42′ 58″.

Having ascertained the tangential angle, fix the theodolite at the tangent point A, and lay off the tangential angle, C A D, the chain being extended from the tangent point A to D, in the same range as the visual axis of the telescope; the theodolite being still fixed at A, lay off the angle D A E = C A D, the chain being now extended from D, to meet the visual axis of the telescope at E, and so on to the end of the curve. Care should be taken to drive down some distinctive stump at the tangent point A, so that it may be easily found afterwards. Stumps should, of course, be driven down at the end of each chain in the curve, and at the point of intersection of the tangents at C.

It may happen some obstruction, such as a house, trees, &c., will prevent seeing from the point A further on the curve than E. If such should be the case, remove the theodolite to D, the point preceding E. Then sight first on E, and continue to

lay off the tangential angles as before until the end of the curve is reached.

PROBLEM III.

The radius A M and angle of intersection K C B being given, to find the length of the tangent A C. (See figure 159.)

Solution.—In the right triangle A M C we have $\frac{\text{A C}}{\text{A M}}$ $=$ tan. A M C $=$ tan. $\frac{1}{2}$ K C B.

$$\therefore \text{ Tan.} = \text{R tan. } \tfrac{1}{2}\text{ K C B.}$$

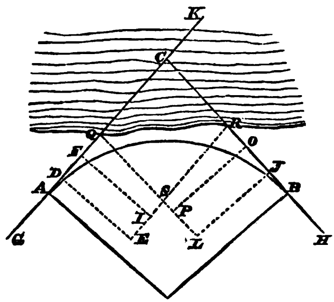
EXAMPLE.

Given K C B $= 22° 52'$, and R $= 80·00$, to find tangent A C.

R $=$ 30·00	log.	3·477121
$\frac{1}{2}$ K C B $=$ 11° 26″	tan.	9·305869
Tan. A C $=$ 606·72		2·782990

PROBLEM IV.

When the point of intersection C of the tangents G A and H B happens to fall in a sheet of water, a wood, or a pile of buildings, to find the angle of intersection K C B.



Set off D E and F I at right angles to G C, and J L and O P at right angles to H C, and let these perpendiculars measure equally, so that lines produced through the points E, I, L, and P, to intersect the tangents G C and H C, may clear the obstacle in the way as at Q and R. Now measure the angle L S R, which is equal to the angle of intersection K C B. Measuring S Q and S R gives Q C and R C, which added to A Q and B R, respectively give the length of the tangents A C and B C.

PROBLEM V.

Given the angle of intersection K C B $=$ A M B, and the tangential angle C A D, to find the length of the curve. (See figure, p. 159.)

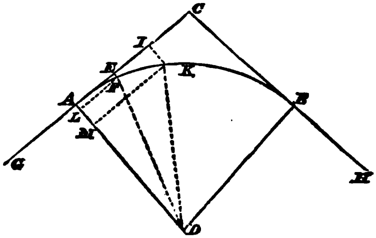
Solution.—Half the angle of intersection, divided by the tangential angle, gives the length of the curve in chains. The length thus found is not the length of the arc, but the length of

the chords. The length of the arc may be found by Rule 1, page 15, Stephenson's "Railway Construction," by Nugent (Weale's Series).

Problem VI.

To lay out a curve on the ground with the chain only, by equidistant offsets from the tangents, no obstruction being on the convex side of the curve.

Let G A and H B be the tangents which are to be connected by a curve, the radius of which is given, and A the point whence the curve is proposed to commence. Produce the tangent G A in the same straight line towards C; measure, say, one chain from A to E. Now, if the length of the offset E F to the curve be known, by raising the offset the proper length, the point F in the curve becomes known.



To find the length of the offset E F. Join F D, and let fall F L perpendicular on A D. In the right triangle F L D, $DF^2 = DL^2 + LF^2$, but $DF = AD$, therefore $AD^2 = DL^2 + LF^2$; transpose LF^2, and $AD^2 - LF^2 = DL^2$; extract the square root of each side, and $\sqrt{AD^2 - LF^2} = DL$. If D L be taken from A D, we get A L = E F, the length of the offset required. The length of the offset I K, at the end of the second chain on the tangent, can be found in a similar manner; for $IK = AM = AD - DM$, D M being equal the $\sqrt{AD^2 - KM^2}$; and so on with regard to any other offset.

EXAMPLE.

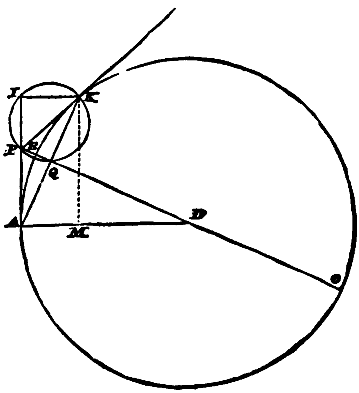
Suppose the radius A D equal 40 chains, to find the length of the offset E F to the curve at the end of the first chain. By the above formula $\sqrt{AD^2 - LF^2} = DL$; that is, $\sqrt{40^2 - 1^2} = DL = \sqrt{1599} = 89{\cdot}987498$; this last number taken from the radius, 40 chains, leaves ·012502 of a chain, the length of the offset E F. If we multiply ·012502 by 792, the number of inches in a chain, we obtain 9·901584 inches = E F. The length of the offset I K is found in a similar manner. For $DM = \sqrt{AD^2 - KM^2} = \sqrt{40^2 - 2^2} = \sqrt{1596} = 89{\cdot}9499687$,

which subtracted from the radius, 40 chains, leaves ·0500318 of a chain. This number, multiplied by 792, gives 39·6247896 inches as the length of the offset I K at the end of the second chain.

So that to obtain the length of an offset from any point on the tangent to the curve, we have only to subtract the square of the distance measured on the tangent from the square of the radius, and extract the square root of the remainder; the root so obtained subtracted from the radius gives the length of the offset.

Setting out a New Tangent.—At every eight or ten chains it may be advisable to recommence operations, which is done by setting out a new tangent to the curve. This may be done by joining the last point found in the curve to the next but one from it in the curve, viz., that corresponding to the offset two chains back. Then a line ranged forward parallel to this chord, and touching the curve at the middle point, will be the tangent required.

The following method will perhaps be found superior to the one just given, as no ground is lost, and affords a better position for ranging the new tangent. Let K be the point whence it is required to start with the new tangent. From A set off on A C, a distance equal to $\frac{A\,I^2 + I\,K^2}{2\,A\,I}$; that is, the sum of the squares of the last offset and the whole distance measured on the tangent, divided by twice the same distance. A line from the point thus obtained, ranged forward through K, the last point found in the curve, will be the tangent required.



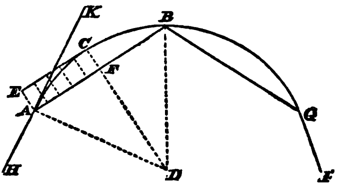
As the formula just given is not only useful in practice, but strictly geometrical, it may be well to give its demonstration. Join A K, and bisect it in Q; connect D Q, and produce it both ways to O and P. Now A K being bisected by the line D Q drawn from the centre, the angles A Q P and P Q K are right angles

(Euclid 3, iii.); it follows that the angles I K Q and I P Q are together equal to two right angles; therefore a circle may be described about the quadrilateral I K Q P. Let the circle be described. Now according to cor. Euclid 36, iii., $K\ A \times A\ Q = I\ A \times A\ P$, but $K\ A \times A\ Q = \frac{K\ A^2}{2} = \frac{A\ I^2 + I\ K^2}{2}$, therefore $I\ A \times A\ P = \frac{A\ I_2 + I\ K^2}{2}$; multiply off by 2, and $2\ I\ A \times A\ P = A\ I^2 + I\ K^2$; divide by $2\ I\ A$, and $A\ P = \frac{A\ I^2 + I\ K^2}{2\ I\ A}$. Now, as A Q and Q P are equal to K Q and Q P, each to each, and the contained angles equal, the line A P is equal to P K; consequently P K is a tangent (Euclid 37, iii.).

Problem VII.

To lay out the curve by offsets from its chord, or chords, when obstructions occur on the convex side of the curve.

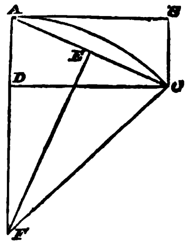
Let A C B be a portion or the whole of a railway curve, H A a tangent at its commencement, C E a tangent at its middle point C, and A D, B D its radii. Take the chord A B, an even number of chains; find the successive offsets corresponding to the radius A D, and the tangent $C\ E = A\ F = \frac{1}{2}\ A\ B$, as in Prob. 6, p. 164. The last offset E A will be equal C F; from C F subtract the successive offsets, and the remainders will be respectively the offsets, which must be set off in an inverted order from A to F, and their order must be again inverted in setting them off from F to B. The operation may be continued by taking other chords, as B G.



Note. This method of laying out a curve may be advantageously used where a winding river, buildings, cliffs, &c., are close to or protrude in some places over the curve. The tangent C E, which is parallel to A B, is not used further than to explain the method of finding the offsets. An important point in relation to this method has been hitherto entirely overlooked by writers on railway curves; that is, they do not furnish any means by which the direction of the chord A B is to be determined. This is necessary. Let the radius of the curve be 40 chains, and the chord A B be 10 chains, how is its direction to be determined? By No. 7 formula for the solution of right triangles, p. 160, the angle A D F is found to be 7° 10′ 50″, but the angle A D F is equal F A K. If a theodolite be fixed at A, and an angle equal to 7° 10′ 50″ be laid off with the line A K, it will determine the direction of the chord A B.

PROBLEM VIII.

Given the radius A F, or the tangential angle t of a curve, and the angle B A C, made by the chord A C with the tangent at A, to find the length of the chord A C.



$$A\,C = \frac{100 \text{ sin. } B\,A\,C}{\text{sin. } t}.$$

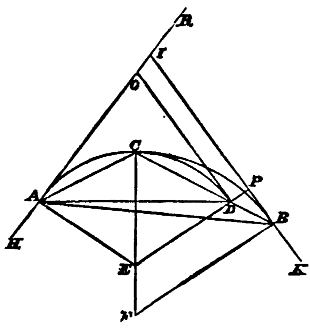
Suppose the curve A C is 4 chains long, and that we wish to find the length of the chord A C. In this case the angle B A C $= 4t$, and $A\,C = \dfrac{100 \text{ sin. } 4\,t}{\text{sin. } t}$, and the radius

$$A\,F = \frac{\frac{1}{2}\,A\,C}{\text{sin. } B\,A\,C}, \text{ or } \frac{\frac{1}{2}\,A\,C}{\text{sin. } 4\,t}.$$

PROBLEM IX.

Given the line A B, which joins the fixed tangential points A and B, the angles B A I and A B I, and the first radius A E of a compound curve, to find the second radius B F to unite the tangents H A and K B.

$$B\,F = A\,E + \frac{B\,D}{2 \text{ sin. } C\,B\,I}.$$



The point D may be determined in the field by laying off the angle I A D $= \frac{1}{2}$(B A I + A B I), and measuring the distance A D $= 2$ A E sin. $\frac{1}{2}$ (B A I + A B I). B D and C B I may then be measured.

When the angle A B I is greater than B A I, that is, when the greater radius is given, the solution is the same, except that the angle D A B $= \frac{1}{2}$(A B I —B A I), and C B I is found by subtracting the supplement of A B D from A B I. We shall also find C B = C D — B D, and therefore

$$B\,F = A\,E - \frac{B\,D}{2 \sin.\ C\,B\,I}.$$

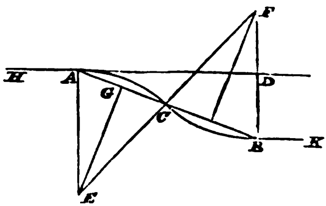
Problem X.

Given the tangents A I and B I, the angle of intersection B I R, and the first radius A E, to find the second radius B F. (See last figure.)

Solution.—Suppose the first curve to be run with the given radius from A to D, where its tangent D O becomes parallel to B I. Through D draw D P parallel to A I, and we have $I\,P = D\,O = A\,O = \overline{\text{Rad. tan. } \tfrac{1}{2}\ B\,I\,R}$ (Prob. 8). Then in the triangle D P B we have $D\,P = I\,O = A\,I - A\,O = A\,I - \overline{\text{Rad. tan. } \tfrac{1}{2}\ B\,I\,R}$, $B\,P = B\,I - I\,P = B\,I - \overline{\text{Rad. tan. } \tfrac{1}{2}\ B\,I\,R}$, and the included angle $D\,P\,B = A\,I\,P = 180° - \tfrac{1}{2}\ B\,I\,R$. In the triangle D P B, the sides D P and P B are given, and the included angle, from which the angle C B I, and the side B D may be computed by formula 14, p. 161. The rest of the solution is the same as the last problem. The determination of the point D in the field is also the same; the angle I A D being in this case $= \tfrac{1}{2}\ B\,I\,R$. When A B I is greater than B A I, that is, when the greater radius is given, the solution is the same, except that $D\,P = \text{Rad. tan. } \tfrac{1}{2}\ B\,I\,R - A\,I$, and $B\,P = \text{Rad. tan. } \tfrac{1}{2}\ B\,I\,R - B\,I$.

Problem XI.

Given the perpendicular distance between two parallel tangents B D, and the distance between the tangent points A B, to determine the reversing point C, and the common radius E C = C F of a serpentine or reversed curve uniting the tangents H A and B K.



$$F\,C, \text{ or } E\,C = \frac{A\,B^2}{4\,B\,D}.$$

If the common radius E C = C F, and the perpendicular distance B D be given, and A B required, then

$$A\,B = 2\sqrt{E\,C \times B\,D}.$$

The reversing point C is the middle point of A B.

Problem XII.

Given the perpendicular distance between two parallel tangents B D, the distance between the two tangents' points A B, and the first radius E C, of a serpentine or reversed curve uniting the tangents H A and B K, to find the chords A C and C B, and the other radius C F.

Draw the perpendiculars E G and F L. Then the right triangles A B D and E A G are similar, since the angle B A D $= \frac{1}{2}$ A E C $=$ A E G. Therefore A B : B D : : E A : A G, or A B : B D : : E C : $\frac{1}{2}$ A C; multiply means and extremes, and

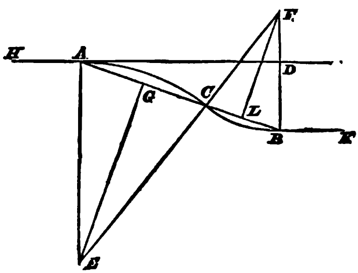
$$A\,B \times \tfrac{1}{2}\,A\,C = B\,D \times E\,C;\text{ multiply off by 2, and}$$

$$AB \times AC = 2\,E\,C \times B\,D.\text{ Divide by } A\,B \text{ and } AC = \frac{2\,E\,C \times B\,D}{A\,B}$$

$$\text{And } A\,B - A\,C = C\,B.$$

To find C F, the similar triangles A B D and F C L give A B : B D : : F C : C L; multiply extremes and means, and F C $\times$ B D $=$ A B $\times$ C L, but C L $= \frac{1}{2}$ C B; therefore F C $\times$ B D $=$ A B $\times \frac{1}{2}$ C B; multiply off by 2, and F C $\times$ 2 B D $=$ A B $\times$ C B; divide by 2 B D, and

$$F\,C = \frac{A\,B \times C\,B}{2\,B\,D}.$$



If B D, A C, and C B are given, to find A B, E C, and C F. We have A B $=$ A C $\times$ C B; by one of the above formulæ A B $\times$ A C $=$ 2 E C $\times$ B D, or 2 B D $\times$ E C, then

$$E\,C = \frac{A\,B \times A\,C}{2\,B\,D}.\quad \text{And } F\,C \text{ or } C\,F = \frac{A\,B \times C\,B}{2\,B\,D}.$$

If E C. C F and B D are given, to find A B, A C, and C B.

$$\frac{A\,B \times A\,C}{2\,B\,D} = E\,C.$$

$\frac{A\,B + C\,B}{2\,B\,D} = C\,F$; add these together, and

$\overline{\frac{A\,B(A\,C + C\,B)}{2\,B\,D}} = E\,C + C\,F$. Multiply off by $2\,B\,D$, and

$A\,B(A\,C + C\,B) = \overline{E\,C + C\,F} \times 2\,B\,D$; but $A\,C + C\,B = A\,B$, therefore

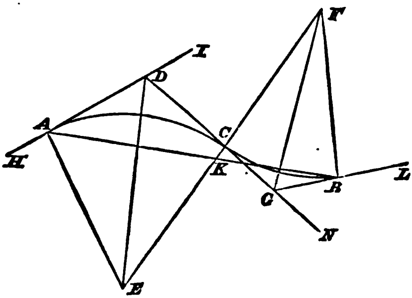
$A\,B^2 = \overline{E\,C + C\,F} \times 2\,B\,D$. Extract the square root of both sides

$A\,B = \sqrt{(E\,C + C\,F) \times 2\,B\,D}$.

Having found A B, we have, as shown above, $A\,C = \frac{2\,E\,C \times B\,D}{A\,B}$.
And $C\,B = A\,B - A\,C$.

Problem XIII.

Given the line A B, which joins the fixed tangent points A and B, the angles D A B and A B G, to find the common radius E C=C F of a serpentine or reversed curve to unite the tangents H A and B L.



1. Formula. $\sin. A\,K\,E = B\,K\,F = \frac{\cos. \frac{1}{2}(D\,A\,B + A\,B\,G)}{\cos. \frac{1}{2}(D\,A\,B - A\,B\,G)}$.

2. Formula. $E\,C = C\,F =$

$$\frac{\frac{1}{2}\,A\,B \sin. A\,K\,E}{(\cos. A\,K\,E - \frac{1}{2}(D\,A\,B + A\,B\,G))\cos. \frac{1}{2}(D\,A\,B - A\,B\,G)}.$$

EXAMPLE.

Given A B = 1500, D A B = 18°, and A B G = 6°, to find E C = C F.

$\frac{1}{2}$(D A B + A B G) = 12°	cos.	9·990404
$\frac{1}{2}$(D A B — A B G) = 6°	cos.	9·997614
A K E 76° 36′ 10″	sin.	9·988018
$\frac{1}{2}$ A B = 750		2·875061
		2·863079

A K E — $\frac{1}{2}$(D A B + A B G) = 64° 36′ 10″ cos. 9·632347

$\frac{1}{2}$(D A B — A B G) = 6° cos. 9·997614

9·629961

E C = C F = Radius = 1710·48   3·233118

Problem XIV.

Given the length of the common tangent D G, and the angles of intersection C D I and B G N, to find the common radius C E = C F of a reversed curve to unite the tangents H A and B L. (See last figure.)

$$C E = C F = \frac{D G \cos. \frac{1}{2} C D I \cos. \frac{1}{2} B G N}{\sin. \frac{1}{2}(C D I + B G N)}.$$

The points A and B are found by measuring from D a distance A D = C E, tan. $\frac{1}{2}$ C D I, and from G a distance B G = C E, tan. $\frac{1}{2}$ B G N.

EXAMPLE.

Given D G = 800, C D I = 14° and B G N = 10°, to find C E = C F.

D G = 800		2·903090
$\frac{1}{2}$ C D I = 7°	cos.	9·996751
$\frac{1}{2}$ B G N = 5°	cos.	9·998344
		2·898185
$\frac{1}{2}$(C D I + B G N) = 12°	sin.	9·317879
Radius C E = C F = 3804·57		3·580306

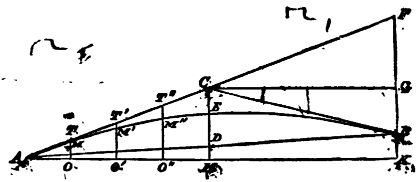
VERTICAL CURVES.

Vertical curves are used to round off the angles formed at the junction of two gradients. Let A C and C B be two gradients meeting at C. The rates of inclination of the gradients are assumed to be known. Thus, starting from A, the rates of inclination of A C and C B may be denoted respectively by r and r'; that is, r denotes what is added to the height at every station on A C, and r' denotes what is added to the height at every station on C B; but as C B is a descending gradient, the quantity r' is therefore minus. By the principles of the parabola we are enabled readily to unite any two gradients by a vertical curve.

PROBLEM XV.

Given r equal the rate of inclination of the gradient A C, and r' the rate of inclination of C B, and the number of stations from A and B, the tangent points to C equal, to unite the tangent points by a parabolic vertical curve.

Let A E B be the parabolic curve required. Through A draw the horizontal line A K; let fall B K perpendicular on A K; produce B K to meet A C produced in F. From C let fall C H perpendicular on A K. Then, since the distance from C to A and B is measured horizontally, A H is equal H K, and therefore A D = D B. The vertical line C D is therefore a diameter of the parabola and the distances T M, T′ M′, &c., to the curve in a vertical direction from the stations on the tangent A C are to each other as the squares of the number of stations from A. That is, if d represent this distance at T, the first station from A, the distance at T′, the second station, would be



$4d$, at the third $9d$, and at B, which is $2n$ stations from A, it would be $4n^2d$; that is, $F B = 4n^2d$, or $d = \frac{F B}{4n^2}$. As F B is still an unknown quantity, it is requisite to find it first in order to find d. Through C draw C G parallel to A K. Then the triangles C F G and A C H are equal, and F G = C H. But C H is the rise in n stations from A to C; that is, $C H = n r$,

or F G$=n\,r$. And G B is the rise in the second gradient C B in n stations; but as r' is minus, G B$=-n\,r'$. Therefore F B$=$F G$+$G B$=n\,r-n\,r'$. If we substitute this value of F B in the foregoing formula for d, we obtain $d=\frac{n\,r-n\,r'}{4n^2}=\frac{r-r'}{4n}$. The value of d being thus found, all the distances d, $4d$, $9d$, $16d$, &c., from the tangent A F to the curve, are known. If T and T' be the first and second stations on the tangent, and the vertical lines T O and T' O' be drawn to the horizontal line A K, the height T O of the first station above A is r, the height T' O' of the second station above A is $2r$, and so on for successive stations we should find the heights $3r$, $4r$, &c.

As we have found T M$=d$, T' M'$=4d$, &c., we shall have for the heights of the curve above the level of A, M O$=$T O$-$T M$=r-d$, T' O'$=$M' O'$-$T' M'$=2r-4d$, and in like manner for the succeeding heights $3r-9d$, $4r-16d$, &c. Then to find the heights for the curve at the successive stations from A, that is, the rise of each height over the preceding height, we must subtract each height from the next following, thus: $(r-d)-0=r-d$, $(2r-4d)-(r-d)=r-3d$, $(3r-9d)-(2r-4d)=r-5d$, $(4r-16d)-(3r-9d)=r-7d$, &c. The heights of the successive stations for the vertical curve are therefore

$$r-d,\ r-3d,\ r-5d,\ r-7d,\ \&c.$$

In finding these heights close attention should be given to the algebraic signs.

EXAMPLE.

Let the number of stations on each side of A C be 4, and A C ascend ·8 per station, and C B descend ·6 per station. In this case $n=4$, $r=\cdot 8$, and $r'=-\cdot 6$. Then $d=\frac{r-r'}{4n}=\frac{8-(-6)}{4\times 4}=\frac{1\cdot 4}{16}=\cdot 0875$, and the heights from A to B are—

$$
\begin{aligned}
r-\ \ d&=\cdot 8-\ \cdot 0875=\ \ \cdot 7125\\
r-\ 3d&=\cdot 8-\ \cdot 2625=\ \ \cdot 5375\\
r-\ 5d&=\cdot 8-\ \cdot 4375=\ \ \cdot 3625\\
r-\ 7d&=\cdot 8-\ \cdot 6125=\ \ \cdot 1875\\
r-\ 9d&=\cdot 8-\ \cdot 7875=\ \ \cdot 0125\\
r-11d&=\cdot 8-\ \cdot 9625=-\cdot 1625\\
r-13d&=\cdot 8-1\cdot 1375=-\cdot 3375\\
r-15d&=\cdot 8-1\cdot 3125=-\cdot 5125
\end{aligned}
$$

TURNOUTS AND CROSSINGS.

The ordinary mode of turning off from a main track is switching a pair of rails in the main track, and laying a turnout curve tangent to the switched rails, placing a frog where the outer rail of the turnout crosses the rail of the main track. One of the rails of the main track switched is represented by A B; B F represents the outer rail of the turnout curve, tangent to A B; and F shows the position of the frog. The switch angle is D A B, formed by the switched rail A B with A D, its former position in the main track. G F M is the frog angle made by the crossing rails, the direction of the turnout rail at F being the tangent F M at that point. The gauge of the track D C, and the range D B of the switched rail A B, are supposed to be known. The switch angle D A B, the sine of which by formula 1, p. 160, is equal D B divided by the length of the switched rail. If the switched rail should be 20 feet long, and D B = ·5 of a foot, then the angle D A B = 1° 41′.

PROBLEM XVI.

Given the radius B E of the centre line of a turnout, to find the frog angle G F M and the chord B F.

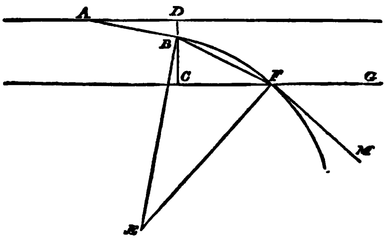
$$\cos. G F M = \cos. D A B - \frac{D C - D B}{E B + \frac{1}{2} D C}.$$

By this formula G F M may be found by a table of natural cosines.

$$B F = \frac{D C - D B}{\sin. \frac{1}{2}(G F M + D A B)}.$$

EXAMPLE.

Given D C, the width of the gauge, equal 4 feet 8½ inches, or 4·708 feet, D B equal half a foot, or ·5′, the angle D A B



= 1° 41′, and the radius B E 500, to find the angle G F M = and the chord B F.

nat. cos. D A B $=$ ·99957 D C $-$ D B $=$ 4·208′

E B $+\frac{1}{2}$ D C $=$ 502·354, and 4·208 $\div$ 502·354 $=$ ·0083765.

Therefore nat. cos. G F M $=$ ·99957 $-$ ·0083765 $=$ ·99119 $=$ 7° 37′ 15″.

To find the chord B F :—

D C $-$ D B $=$ 4·208	log. 0·624076
$\frac{1}{2}$ (G F M $+$ D A B) $=$ 4° 39′ 7″	sin. 8·909033
B F $=$ 51·873	1·715043

Problem XVII.

Given the frog angle G F M, to find the radius B E of the centre line of a turnout, and the chord B F. (See last figure.)

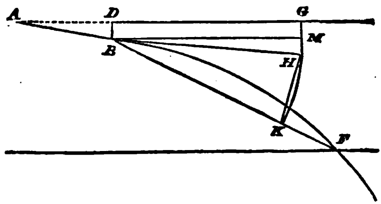
$$B\,E = \frac{\frac{1}{2}(D\,C - D\,B)}{\sin.\frac{1}{2}(G\,F\,M + D\,A\,B)\,\sin.\frac{1}{2}(G\,F\,M - D\,A\,B)} - \frac{1}{2}D\,C.$$

$$B\,F = \frac{D\,C - D\,B}{\sin.\frac{1}{2}(G\,F\,M + D\,A\,B)}.$$

Problem XVIII.

To find the position of a given frog practically on the ground.

Let L $=$ length of rail, f $=$ length of frog, and w its width. From B as a centre, with a radius B H $=$ 2 L, describe the arc



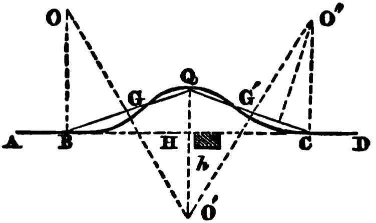
G H K, and from the inside of the rail at G measure G H $=$ 2 D B, and from H measure H K, such that H K : B H :: $\frac{1}{2}w : f$, or H K : 2 L :: $\frac{1}{2}w : f$; multiply means and extremes, and H K $\times f =$ L w; divide by f, and H K $= \frac{L\,w}{f}$. Then a straight line through B and the point K will strike the inside of the other rail at F, the place for the point of the frog.

If the turnout is to reverse and become parallel to the main track, or enters a parallel main track by a second switch, and becomes a crossing, the problems on reversed curves already given will generally be found sufficient.

Problem XIX.

To make a given deviation from a straight line of railway, by three curves; that the works of the line may avoid a building or other obstruction, situated on or near to it.

1. Let A B C D be a straight portion of the railway, *h* a building or other obstruction on the line Take H Q of a sufficient length for a deviation, that the lateral works of the line may avoid the object at *h*; and through Q draw a curve GQG′ of radius QO′ = to, or greater than 1 mile. Draw also, two other curves, BG, G′C, of like radius, to touch the first curve at G and G′, and the line at B and C: then the lines OO′, O″O′, joining the centres of the curves, will pass through their points of contrary flexure at G and G′.



2. *Calculation.* Put r = common radius OB = O′Q = O″C, and d = required deviation = H Q; then B H = H C = $\sqrt{d(4r - d)}$, and the four equal chords B G, G Q, &c., are each = $\sqrt{dr}$.

EXAMPLE.

Let the deviation Q H = d = 3 chains, and the radius B O = r = 1 mile = 80 chains; then BH = HC = $\sqrt{3(320-3)}$ = 30·84 chains, and B G = G Q = &c. = $\sqrt{240}$ = 15·49 chains. These distances being set out will give the required points in the deviation curve B G Q G′ C, as required.

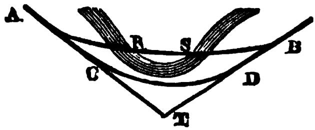
Note. These deviations are now more frequently made than formerly by some of our most eminent engineers to avoid large cuttings and embankments; thus contributing greatly to the economy of construction, especially of branch lines where great velocities of transit are not required.

EXAMPLES OF EXPENSIVE SEVERANCE OF PROPERTY BY IMPROPERLY SETTING OUT RAILWAY CURVES.

1. It has been already shewn (Prob. I.) that the curve adopted in joining two straight portions of a railway, ought to avoid, as far as possible, expensive cutting, severance, bridges, &c., provided its radius be equal to or greater than one mile, if other considerations do not require it to be less. The following

injudicious violations of this rule came immediately under the author's observation.

In the annexed figure, A R S B is a curve of 1½ miles radius, at Westwick, near Cambridge, in the Wisbeach, St. Ives, and Cambridge Railway: this curve unnecessarily makes an expensive severance, by being laid out through a gentleman's park, within 100 yards of his house, and by crossing and recrossing a brook 16 feet in width at R and S, within the space of a few chains, the brook being left by the curve only 30 yards, at the widest part between the points of crossing. Thus the



line here requires two oblique bridges at R and S, which might have been avoided, as well as the expensive severance of the park, by extending the tangential portions A C, B D of the railway a few chains each, and substituting a curve C D of one mile radius to supply the place of the one thus ignorantly adopted; the ground being almost perfectly level, and the limit of deviation at the same time, admitting, nay at once suggesting, this improvement of the line, as it respects expense of construction.

2. A similarly expensive severance was unnecessarily made in the same line at Histon, about three miles from Cambridge, by laying out a curve of 3 miles radius, through cottages and gardens, all of which (being situated within three chains of the middle of the line on the north side thereof,) would have to be purchased by the railway company; besides the curve at the same place crossed two public roads within 40 yards of their junction: but had the straight portions of the line been extended at both ends of the curve, as in the former case, and a curve of 1 or 1½ miles radius been employed, instead of the 3 mile radius, both the cottages and gardens might have been passed near their southern boundary, and only one road would be required to be crossed, at a short distance from the junction of the two roads in question, the limits of deviation and the ground (being perfectly level) at once suggesting this improvement to any engineer of ordinary skill.

There was a slight hope that the head engineer of this line would have rectified these expensive blunders, previous to the formation of the railway: although such important matters were too often overlooked or disregarded in the *Hudsonian*

hurry of railway projects; which, I understand, was the case on this occasion.

THE PRACTICE OF ENGINEERS IN THE ADOPTION OF CURVES IN VARIOUS RAILWAYS.

The practice of engineers differs very widely in the adoption of curves, some choosing curves of large radius, at great sacrifices of cost, and others adopting very small ones to avoid expensive cuttings, embankments, &c.,

On the Great Western Railway, "the curves are in general very slight, chiefly of 4, 5, and 6 miles radius. Mr. Brunel considered that even a mile radius is not desirable, except at the entrance of a depôt, where the speed of the engines is always greatly slackened. And, except in these instances, the only deviation from this rule, which he has admitted, is in the curve, about one-fourth of a mile below one of the inclines, where the radius is three-fourths of a mile."—*Railway Magazine*, vol. I., page 418.

Mr. R. Stephenson, in his evidence on the projected Brighton Railway in 1836, stated that no curve had a less radius than 1½ miles, in the line he proposed, which he considered a most convenient radius for the high velocities required for passenger trains. Mr. Stephenson does not, however, limit the *minimum* curvatures to three-fourths of a mile, if engineering difficulties, or other considerations of a sufficiently important character, suggest the adoption of curves of smaller radius.

On the Chester and Birkenhead, Birmingham and Derby, Edinburgh and Glasgow, Arbroath and Forfar, and many others, the *minimum* radius adopted for the curves is 1 mile; and the same radius is also the *minimum* in the Birmingham and Gloucester, and the Sheffield and Manchester lines, which are curved throughout almost the whole of their lengths. The London and Birmingham line, though constructed through a very uneven country, has chiefly curves of a radius exceeding 1 mile; while the Manchester and Leeds line has curves generally of three-fourths of a mile radius, with a few considerably less.

A still greater deviation from the *minimum* limit of one mile for the radius of curves, will be found in railways where great engineering difficulties were to be encountered, especially in mineral lines. The Taff Vale Railway, which is a single line, appears from Sir F. Smith's report to have curves of the subjoined radii and length.

Chains radius.		Length of curve.
7	....	7 chains.
10	....	26 ———
11	....	7 ———
12	....	18 ———
15	several curves ...	201 ———
20	ditto...............	182 ———
22	ditto...............	173 ———
25	....	29 ———
26	....	37 ———
28	....	21 ———
30	....	88 ———
40	....	85 ———
60	....	20 ———
80	....	120 ———

These curves were used to avoid the repeated crossing and recrossing the river Taff, and to avoid the formation of several lofty embankments. The same considerations have led to the adoption of curves of similarly small radius in other mineral lines.

Pambour's formula may be successfully adopted to assign a proper super-elevation to the exterior rail to counteract the centrifugal force, arising from high velocities of trains in curves of small radius. This formula, with the results deducible therefrom, shall be given in the last chapter of this work.

CARELESS EXPENDITURE IN THE CONSTRUCTION OF RAILWAYS, FROM THE NON-ADOPTION OF CURVES, FROM IMPROPERLY LAYING OUT GRADIENTS, EXACTIONS OF LAND-OWNERS, &c.

In addition to the expensive and unnecessary severance of property by improperly setting out railway curves, of which examples, are given at page 175, similar wasteful expenditure has resulted from the non-adoption of curves in numerous instances. It has been shewn in Prob. XII. that a lateral deviation may be made in a straight line of railway to avoid buildings or other expensive property, deep cuttings, or other engineering difficulties. This precaution, in a vast majority of instances, has not been adopted, as may be seen in the construction of a great many of the more early railways; and much unnecessary expense has thus been incurred.

Also, where the geological character of the country through which the railway passes, differs considerably, presenting material for excavation throughout the length of the line, varying from loose sand to hard rock, and *vice versa;* through such varying

materials the facility or difficulty of excavation must be especially considered in laying out the gradients; larger excavations must be adopted in the loose material, and smaller ones in stiff clays, hard rocks, &c., than would have been adopted, had no such variety in the strata existed. *Considerable economy in the construction of a railway will result by judiciously taking into account all these circumstances.* These important subjects, in the railway mania of 1845, were by far too little, or not at all considered, partly through pressing too much business on a few of the chief engineers, by which they were compelled to confide these important works to unskilful apprentices and other incompetent persons, and partly through the ignorance or wilful negligence of others, who had the audacity to put themselves forth, and were accepted by the public as chief engineers at that time, by which a worse than useless expenditure of several millions of the money of the shareholders in these projects was incurred, their blunders being now obvious even to illiterate agricultural labourers.

CHAPTER III.

ON SETTING OUT THE SURFACE WIDTHS OF RAILWAYS.

After the centre of a railway has been marked out, as directed in Chap. II., the line must again be carefully levelled, and the stumps that mark the line must be numbered and entered consecutively in the level book in a vertical column, with the corresponding depths of cuttings or embankments in a second column (see Level Book, p. 187); these depths are estimated from the formation level, which is commonly about 2 feet below the intended line of the rails; the 2 feet are afterwards to be filled up with gravel to form the permanent way.

The line is now prepared for setting out the surface widths, the simplest case of which is when the surface of the ground is level as well as coincident with the formation level of the intended railway. In this case it is only required to set out half the width of the formation level on each side of the centre stump, perpendicular to the direction of the railway, adding to each half width the intended width of the side fence, and putting down stumps to mark the half widths and breadths of the fences.—When the surface of the ground is above or below the formation level, which is commonly the case, the widths must be set out by the following Problems:—

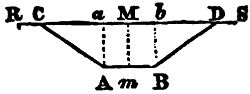
Problem I.

To set out the width of a railway cutting, when the surface of the ground is laterally level, and at a given height above the formation level, the ratio of the slopes being given.*

In the annexed cross section of the cutting, R S is the horizontal surface of the ground; A B the formation level;

* The ratio of the slopes is the proportion that the batter C *a* bears to the depth A *a*. When this ratio is as 1 to 1, C *a* = A *a*; when it is as $1\frac{1}{2}$ to 1, C *a* = $1\frac{1}{2}$ times A *a*. This ratio varies according to the nature of the material through which the cutting is made, being less in rocky or clayey ground, and greater in soft or sandy ground.

A C, B D the side slopes; M the middle stump, and $Mm = Aa = Bb$ the perpendicular depth of the cutting. Multiply the depth Mm by the ratio of the slopes, to which add the half width Am or aM, the sum is half the surface width of be set out from M to C; after which set out R C for the width of the fence. The same operation must be repeated on the other side of M.



EXAMPLE.

Let the width of the formation level AB = 33 feet, the depth Mm of the cutting 30, the ratio of the slopes as $1\frac{1}{2}$ to 1, and the width of the side fences each 6 feet; required the width of the cutting, and the width of land included by the fences.

$30 \times 1\frac{1}{2} + \frac{33}{2} = 45 + 16\frac{1}{2} = 61\frac{1}{2}$ feet $= MC = MD = \frac{1}{2}$ width of cutting; and $61\frac{1}{2} + 6 = 67\frac{1}{2} = MR = MS = \frac{1}{2}$ width of land.

The double of this is the whole width.

Construction of the cross section. Draw A B = width of formation level = 33 feet; perpendicular to A B, at its middle point m, draw Mm = given depth = 30 feet; through M parallel to AB draw CD, making CM, MD each $= \frac{1}{2}AB + 1\frac{1}{2} \times Mm$; join AC, BD; then ABDC is the cross section required.

NOTE 1. The numbers in the column, marked "computed half-widths" in the level book are found by this Problem.

NOTE 2. If the figure ABDC be inverted, it will represent the cross section of an embankment; for setting out the width of which the same method obviously applies, as that just given for a cutting.

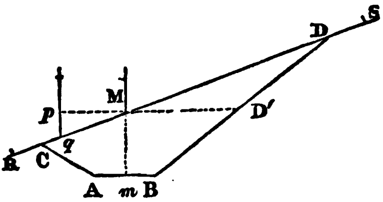
NOTE 3. Let $\omega = AB$ = breadth of the formation level, $d = Mm$ = depth of cutting or embankment, and r = ratio of the slopes. Then $1 : r :: d : dr = aC$; hence $dr + \frac{1}{2}\omega = MC = MD$; $2(dr + \frac{1}{2}\omega) = 2dr + \omega = CD$ = surface width of cutting or embankment; and $\frac{1}{2}(AB + CD) \times Mm = (dr + \omega)d$ = area of the cross section.

PROBLEM II.

The same things being given as in the last Problem, to set out the width of the cutting, when the ground is laterally sloping, the lateral fall of the ground in a given horizontal distance being also given.

Let C D be the sloping surface of the ground, A B D C the cross section of the cutting, and $p D'$ a horizontal line passing through the centre stump, M, MD' being the computed half width of the cutting.—Fix the levelling instrument so that by turning the telescope 2, 3 or more chains of the line may be

seen, on both sides of it; set up a levelling staff at M, and another at q, not exceeding a chain's distance from M, observing the level readings on both staves, the difference of which is equal to pq; measure the sloping and horizontal distances Mq, Mp with a tape line in feet. Then take the computed half width M D′, (found by the last Problem) and multiply it by M q, reserving the product; multiply the difference of the stave readings $p\,q$ by the ratio of the slopes, add and subtract the product to and from the horizontal distance M p, reserving the sum and difference; lastly, the reserved product, being divided by the reserved sum, will give the corrected half width M C, and by the reserved difference the corrected half width M D.



EXAMPLE.

The depth of the cutting at M is 22 feet, the bottom width A B = 36, the sloping distance M q = 25, the level distance M p = 24, the difference of the readings of the staves $p\,q$ = 7 feet, and the ratio of the slopes as $1\frac{1}{2}$ to 1. What are the corrected half widths M C and M D?

$22 \times 1\frac{1}{2} + \frac{36}{2} = 33 + 18 = 51$ feet = computed half width
25

1275 reserved product.

24
$7 \times 1\frac{1}{2} = 10\frac{1}{2}$

reserved sum = $34\frac{1}{2}$) 1275 (36·95 feet = cor. $\frac{1}{2}$ width M C.

reserved diff. = $13\frac{1}{2}$) 1275 (94·44 feet = cor. $\frac{1}{2}$ width M D.

Construction of the cross section.—Draw B D′ M m, as in Prob. I.; lay off the given horizontal distance M p = 24 feet; draw $p\,q$ parallel to M m, and equal to the difference of the stave readings at M and q; through M, q draw C D, meeting A C in C and B D′ prolonged in D; then A B D C is the required cross section.

NOTE 1. The following general formula for the values of M C, M D, is easily remembered, and would perhaps be preferred to the rule, as given above, by those who are accustomed to the use of symbols.

$$\text{Corrected } \tfrac{1}{2} \text{ width} = \frac{\omega' s}{l \pm r h}.$$

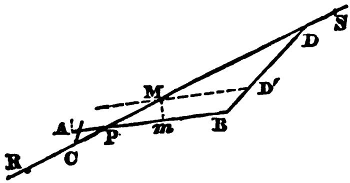
The positive sign is used for M C, and the negative for M D; M D′ being $= \omega'$, $p q = h$, $M q = s$, $M p = l$, and the ratio of the slopes as $r : 1$. The investigation of this formula is given in *Baker's Railway Engineering*, page 35, wherein, when h is very small, s may be taken $= l$ without error.

NOTE 2. If the figure A B D C be inverted, it will obviously represent the cross section of an embankment of like dimensions, the longer distance, in this case, being measured down, and the shorter up the slope.

PROBLEM III.

The same things being given, as in Prob. II., *to set out the width, when it consists partly of a cutting and partly of an embankment.*

In the annexed figure B D P C A is the cross section of the works of a railway, consisting of the cutting B P D and the embankment A P C; A B is the formation level, M the central stump, and D C the sloping surface of the ground. When the cutting B P D extends over more than half the formation level A B, the corrected half width M D is found as in Prob. II. But to find the other corrected half width MC, passing under the embankment A P C′, proceed as follows. Multiply M D by the difference of the width of the formation level A B and the estimated half width MD′, and divide the product by MD′, and the result will be the corrected half width M C.



EXAMPLE.

Let the bottom width AB = 36 feet, the depth Mm = 4 feet, the ratio of the slopes as 2 : 1, and the difference of level readings 7 feet at 25 and 24 feet from the central stump, respectively estimated on the slope and horizontally; required the corrected half width M D, M C.

By Prob. I.... $4 \times 2 + \dfrac{36}{2} = 26$ feet = estimated $\tfrac{1}{2}$ width MD′.

By Prob. II. . $\dfrac{26 \times 25}{24-(7+2)} = 65$ feet = corrected $\tfrac{1}{2}$ width M D.

By Prob. III. $\dfrac{65 \times (36-26)}{26} = 25$ ft. = corrected $\tfrac{1}{2}$ width M C.

Construction of the cross section.—Proceed as in Problem II., excepting that A C must here be drawn parallel to B D.

NOTE 1. The formula for finding the corrected half-width, where there is both a cutting and an enbankment is

$$\text{M C} = \frac{(\omega - \omega')s}{l - rh},$$

wherein ω is the width of formation level, and the other symbols the same as in Note 1, Prob. II. See *Baker's Railway Engineering.*

NOTE 2. To find the distance from M to P, where the cutting and embankment meet, use the following proportion;

$$\text{as } pq : \text{M}q :: \text{M}m : \text{M P},$$

$$\text{that is, } 7 : 25 :: 4 : \frac{25 \times 4}{7} = 14{\cdot}28 \text{ feet} = \text{M P}.$$

$$\text{or the formula } \frac{ds}{h} = \text{M P}.$$

NOTE 3. When the sloping surface of the cutting passes through the middle point of the formation level, that is, when the points M, m, and P, coincide; then $\omega' = \frac{1}{2}\omega$, and the formula in Note 1, becomes

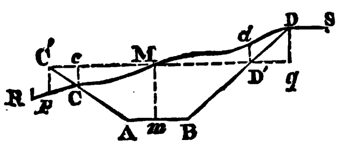
$$\text{M C} = \text{M D} = \frac{\frac{1}{2}\omega s}{l - rh}.$$

In this case the cutting and embankment are equal.

NOTE 4. By inverting the cross section of this Problem, it will at once be seen that a like calculation will be required, in the case of APC being a cutting, and B P D an embankment.

PROBLEM IV.

To find the surface width of a cutting where the ground is very uneven.



The annexed figure is a cross section of a cutting, wherein the surface CMD of the ground is very uneven. The method of solving this Problem is by approximation, and will be best shown by an example in numbers.

Let AB = 36 feet, Mm = 32, and the ratio of the slopes as 2 : 1; then MC′ = MD′ = (32 × 2) + (½ × 36) = 82 feet; lay off this distance horizontally from M to d; then d is directly above D′. Observe the difference of level readings at M and d, which in this case is 9 feet; which being multiplied by the ratio of the slopes, that is by 2, gives 18 feet = approximate distance Dd; whence MD = Md + dD = 82 + 18 = 100 feet. Again, place a level staff at D, and the reading will be found to

be 9.7 feet greater than that at M, or $9{\cdot}7-9=0{\cdot}7$ feet greater than at d; therefore the place of the point D requires further correction, which is thus effected; $0{\cdot}7\times2=1{\cdot}4$ feet = second correction; whence $MD=100+1{\cdot}4=101{\cdot}4$ feet; which, as the latter correction is small, may be safely assumed to be the true distance of D from M, or the horizontal distance M q. The method of finding the other corrected half width MC = horizontal distance M c, is the same as that just given, excepting that the repeated corrections are subtracted from the computed $\frac{1}{2}$ width instead of being added thereto. In this manner the horizontal distance of C from M is found to be 61·8 feet.

Construction of the cross section.—Draw A B D′ C′, as in the preceding Problems; on C′ M D′, as a *datum* line lay off the reduced levels of the several undulations of the surface C M d D of the ground; (see Chap. I., p. 148,) then A B D C is the section required.

NOTE 1. When the differences of the levels at M, d and D are very great, it will require three, four, or more approximations, similar to those just given, to each of the corrected half widths.

NOTE 2. This cross section may be inverted for an embankment, as in the preceding Problems.

LEVEL BOOK.

A level book of the following form is used in setting out the cuttings and embankments of railways.

	No. of Stumps.	Depths of Cuttings or Embankments.	Computed Half Widths.	Corrected Half Widths to Edge of Cuttings of Foot of Embankment. North.	Corrected Half Widths to Edge of Cuttings of Foot of Embankment. South.	Whole Widths including Fences, each 6 feet in Width.
		feet.	feet.	feet.	feet.	feet.
cutting.	183	28·00	57·00	61·80	101·40	175·20
cutting.	184	4·00	21·00	22 63	48·37	83·00
cutting.	185	30·60	60·90	64·77	71·32	148 09
embankment.	186	2·16	18·24	18·00	19·44	49·44
embankment.	187	16·08	39·12	40·68	58·24	110·92
embankment.	188	20·00	30·00	36·16	48 00	96·16

NOTE. The depths in the second column, are found by calculation. (See Levelling, p. 154,) or by careful measurement from the sections; but the latter method is the less correct of the two. The computed half widths, in the 3rd column, are found by Prob. I.; the corrected half widths, in the 4th and 5th column, by Probs. II., III., and IV., and the widths in the last column are the sums of these in the 4th and 5th *plus* the breadths of the two side fences of the railway.

PROBLEM V.

To calculate the quantity of land for a projected railway.

In preparing the estimates for a projected railway, the required quantity of land is commonly found, without respect to the lateral sloping of the surface of the ground, by taking considerable lengths of regularly rising or falling ground, in one calculation, the depths of the ends of such lengths being measured for the purpose with the vertical scale.

RULE.—Find the surface widths, fences included, from the given depths, at each end of the given length, by Prob. I.; multiply their sum by the length in chains, and divide the product by 1320 for the product in acres.

EXAMPLE.

Let the length be 16 chains, the depth of the cross sections at the ends 18 and 58 feet, the width of the formation level 36 feet, the ratio of the slopes as 2 to 1, and the width of the fences 6 feet each; required the area of the surface.

By Prob. I.

$$36 + (18 \times 4) + (6 \times 2) = 120 = \text{width of one end,}$$
$$36 + (58 \times 4) + (6 \times 2) = 280 = \text{width of the other,}$$

$$\begin{array}{r} \overline{400} = \text{sum of widths.} \\ 16 \\ \hline 132{\cdot}0)640{\cdot}0 \\ \hline \end{array}$$

The content $= 4{\cdot}84848 = 4a.\ 3r.\ 16p.$

Or, by putting w' and w'' = computed $\frac{1}{2}$ widths of cuttings, as found by Prob. I., l = length, and f = breadth of one of the fences, there will result,

$$\frac{(w' + w'' + 2f) \times l}{660} = \frac{(54 + 134 + 12) \times 16}{660} = 4{\cdot}84848 \text{ acres, as above.}$$

PROBLEM VI.

To find the exact quantity of land for a railway.

RULE.—Take the widths, at the end of every chain, from the last column of the level book; add continually together the first and last widths, with twice the sum of all the intermediate widths, and divide the result by 1320 for the content in acres.

EXAMPLE.

Required the content corresponding to the several widths, in the preceding level book.

$$
\begin{array}{rrl}
 & 83{\cdot}00 & \\
 & 148{\cdot}09 & \\
 & 49{\cdot}44 & \\
 & 110{\cdot}92 & \\
\hline
 & 391{\cdot}45 & \\
 & 2 & \\
\hline
 & 782{\cdot}90 & = \text{twice sum of intermediate widths,} \\
 & 175{\cdot}20 & = \text{first width,} \\
 & 96{\cdot}16 & = \text{last width.} \\
\hline
1320 \left\{ \begin{array}{l} 12 \\ 11{\cdot}0 \end{array} \right. & \left| \begin{array}{r} 1054{\cdot}26 \\ \hline 87{\cdot}855 \end{array} \right. & \\
\hline
 & {\cdot}79868 & = 0a.\ 3r.\ 8p. \text{ the content.}
\end{array}
$$

NOTE. It is usual in practice to find the contents of the ground required for the works of a railway from the several proprietors, by measurement from the two-chain plan, prepared for the use of the contractors. Copies are also taken from the plan, showing the positions and extents of ground required from the several proprietors for the works of the railway.

CHAPTER IV.

TUNNELLING.

1. When the depth of a railway cutting reaches 60 feet, and the ground afterwards rises rapidly for a considerable distance, the further progress of the works of the line will be the most economically conducted by making a tunnel: previous to the setting out the earthwork of which, the ground, under which it passes, must be again levelled with great care; and, if the tunnel pass beneath a very high summit, the levelling operation must be checked by the method given in Chap. I., p. 156: for, if there be the most trifling inaccuracy in the section, the gradient or gradients, on which the tunnel is proposed to be formed, will not meet at the points shewn on the section, thus greatly embarrassing the mining operation. See the tunnel, Plate III.

2. If the tunnel be formed on a single straight gradient, the

gradient must be so arranged as to incline to one of its extremities, and in order to prevent the accumulation of water in the tunnel. Strong straight poles must be firmly and perpendicularly fixed on the surface of the ground, in the proposed direction of the tunnel, one of which must be at the summit, from whence the direction of both ends of the tunnel may be observed, that the shafts from the surface to the tunnel may be sunk in the true direction. The shafts are usually sunk at the distance of four or five chains from one another, for the purpose of ventilating the tunnel, as well as for drawing the earth, &c. out of it, and for checking the accuracy of the work, during the mining operation, the depths of the shafts being determined by measurement from the section.

3. If the tunnel be a long one, and in springy ground, it would be advisable, if convenient, to form it upon two gradients, inclining to the extremities of the tunnel, as this arrangement would contribute much to the liberation of the water, during the mining operation, which is usually commenced at both ends of the tunnel at the same time,—but, if the work of the tunnel be required to be also commenced at the bottom of the shafts, as soon as they are sunk, which is sometimes the case, the accumulated water must be drawn from the shafts, or a head-way must be driven from the extremities of the tunnel to the shafts to take off the water.—A head-way being only 5 feet in height and 3 in width, may be more readily formed than the more extensive works of a tunnel, and will be found the most efficient method of draining a tunnel in a wet or springy ground, and thus greatly to facilitate the mining operation.

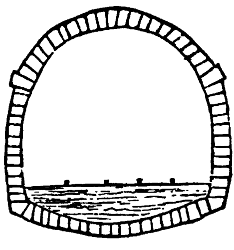
4. When the tunnel is required to have a curve, through a part or the whole of its direction, the curve must be carefully laid out on the surface of the ground, by one or other of the methods given in Chap. II., proper allowances being made for acclivities and declivities, and strong poles being fixed in the whole direction of the tunnel, as pointed out in Art. (2.), that the shafts may be sunk so as accurately to meet the works of the tunnel below, that their true direction may be secured, this will be more especially necessary where the tunnel is curved, though curves ought, if convenient, to be avoided in tunnels, as accidents, attributable to curves, are more dangerous in tunnels, there being less chance for escape, while assistance cannot be so efficiently given in such secluded situations.

5. If the mining operations of the tunnel be required to commence at the bottoms of the shafts, before the head-ways have been driven to them from the ends of it, the angle that the

magnetic meredian makes with the direction of the tunnel must be carefully observed at the top of each shaft, with a miner's compass, (a kind of circumferentor,) that the proper direction of the work may be set out with the same instrument at the bottom of the shaft: but, if the tunnel be curved, the direction of the tangent to the curve at the top of each shaft must be observed, that the same direction may be given to it at the bottom of the shaft, and that from thence the curve may be laid out in the tunnel, in the same direction as it was on the surface of the ground; it would be also advisable to bore from the surface to the tunnel, before the works thereof have proceeded far, to test their accuracy, as ferruginous matter in the earth might cause the magnetic needle to deviate, and thus give a wrong direction to the works, if this precaution be not taken.

6. The excavations of a tunnel on the narrow gauge should be about 30 feet in depth and width, the depth extending 5 or 6 feet below the intended line of the rails, to give room for the inverted arch and the ballasting: but when the tunnel is made through rock sufficiently hard to form the side walls, its height need not exceed 22 or 25 feet, and its width 26 feet, the excavation in this case extending only to the formation level. For the railways on the broad guage the excavations of the tunnel must be proportionably larger.

The cross-section of the masonry of tunnel is shown in the annexed figure, with the ballasting on which the rails are laid. This cross-section is such as is required where the excavation of the tunnel is made through loose earth, the arch above being only requisite when made through hard rock.—The following notes on the construction of tunnels, are chiefly extracted from *Dempsey's Practical Railway Engineering.*



NOTE 1. Like mining and all other subterranean operations, the construction of a tunnel can be but little aided by mechanical appliances; it chiefly requires hard manual labour, exercised under circumstances which do not admit of that thorough superintendance which promotes economy, and, moreover, liable to unforeseen interruptions, of surmounting which neither the manner nor the expense can be predetermined. Thus the Kilsby tunnel, on the North Western Railway, was estimated to cost £40. per yard lineal; whereas the actual cost was £130 per yard, owing to its intersecting a quicksand that had escaped the trial borings. Thus a vast expense was incurred in setting up and working pumping machinery to dry the sand. The Box tunnel on the Great Western Railway, excavated through oolite rock, and lined with masonry only through a portion of its length, cost upwards of £100. per lineal yard. The

length of this tunnel is 3123 yards, or upwards of 1¾ miles, it has eleven principal shafts, and four intermediate ones. The Bletchingly and Saltwood tunnels in the South Eastern Railway, cost respectively £72. and £118. per lineal yard, the greater cost of the latter work arising from the great body of water in the sand which it intersects.—The method of proceeding with tunnelling depends mainly upon the kind of material to be excavated. This having been generally ascertained by boring and trial shafts, which must be sufficiently capacious to admit readily of lowering men and materials, raising materials excavated, fixing pumps, and also for starting the headway of the intended tunnel, when the required depth is reached.

NOTE 2. The working shafts are made from 8 to 10 feet internal diameter. They are of brick work, usually 9 inches thick, and carried up 8 or 10 feet above the surface of the ground. These, and all other shafts, rest upon curbs of cast iron, fitted into the crown of the tunnel, and forming a level base for the shaft. The air shafts are of a smaller thickness and diameter, the latter of which is usually about 3 feet.—The number of working shafts will depend chiefly on the rate of speed with which the work is required to be accomplished. With plenty of men, horses, material, and plant, the work is much facilitated by sinking extra shafts, which will usually repay their cost.

NOTE 3. The Watford tunnel, 75 chains in length, on the North Western Railway, was worked with six shafts, about 8 feet internal diameter; the brick work was moulded to fit the circumference of the shafts, and laid in two half-brick rings. Air shafts were sunk at about 2½ chains distance on each side of each working shaft. The arch and side of the tunnel were chiefly made two bricks thick, and the invert one and a half brick, except where the stratum, passed through, seemed to suggest an increased or diminished thickness. The form of the top of the tunnel is nearly semi circular, supported by curved side walls standing on stone footings, or skew backs, which rest on the invert.

NOTE 4. In commencing the work of the Saltwood tunnel, referred to in Note 1., great difficulty was encountered from the great quantity of water in the green sand, which the tunnel intersects. The course adopted was to make a headway 5 feet high and 4 wide quite through the hill, on a level with the bottom of the tunnel, in which the water was collected and drained off before the tunnel was begun. The size of this, and also the Bletchingley tunnel, is 24 feet wide at the broadest part, 30 feet including the side walls; 25 feet high in the clear, 30 feet including the invert and top arch, and 21 feet above the level of the rails. The brick work of the top arch and walls varies from 2½ to 4 bricks in thickness; and the invert is 3 bricks in thickness.

NOTE 5. The section of the Box tunnel, already referred to, was designed to be 27½ feet wide at the springing of the invert, and 30 feet wide at a height of 7½ feet above this, and the clear height above the rails 25 feet. As a great portion of the tunnel is constructed by mere excavation, and without masonry, these dimensions are, in some cases, departed from, in order to clear away loose portions of the stone, and secure solid surfaces. Where brick work is used, the sides are seven half brick rings in thickness, the arch six, and the invert four. During the construction, the constant flow of water into the works, from the numerous fissures of the rock, compelled pumping on a most expensive scale to be adopted.

NOTE 6. When much water issues from the earth, in sinking shafts or building tunnels, the back of the wall should be well lined with puddle, and Roman cement should be used instead of mortar. The entire tunnel at Kilsby was built with Roman cement, the thickness of the brick-work being mostly 27 inches. This tunnel is 2423 yards long: it has two ventilating shafts 60 feet

in diameter; the brick-work of which is 3 feet thick, and laid in Roman cement throughout: these shafts intersect the line of the tunnel, and form curved recesses, by part of their width extending beyond the tunnel on both sides.

CHAPTER V.

THE METHODS OF FINDING THE CONTENTS OF RAILWAY CUTTINGS, &c.

In making the estimates for a projected railway, the contents of the several cuttings, embankments, &c., are in most cases found by tables, calculated for the purpose, the surface of the ground being considered as on the same level as the centre of the line. But when the projectors of the line have been empowered to construct it, cross sections of the cuttings are carefully taken, at every variation of the surface of the ground, especially if the surface be sidelaying, or inclined laterally with respect to the direction of the line, as in the cross sections in Problems II., III., and IV., Chap. III. The distance of these cross sections may vary from 10 or 12 chains to less than 1 chain, according to the regularity or irregularity of the slopes of the surface. These cross sections must next be plotted on a large scale, as in the problems just referred to, and their areas found by any of the methods given in Chap. III., preparatory to finding the contents of the cuttings by tables; but if the surface lines of any of the cross sections be level, or so nearly so as to be readily reduceable to a level surface line by casting, their areas need not be found, their depth only being required for finding the contents by the tables.

NOTE. Some take a mean of the areas of every two consecutive cross sections, and others a mean of the depths, where the surface line is level, as a basis for calculating the contents of the cuttings, *which methods are both erroneous, especially where the consecutive areas or depths of the cross section differ considerably.*

TABLES FOR FINDING THE CONTENTS OF RAILWAY CUTTINGS, &c.

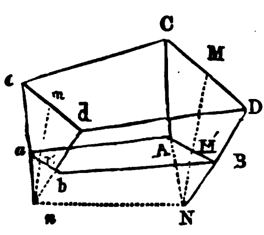
Numerous tables exist for this purpose, some of which are voluminous; those by M'Neill, Bidder, Huntingdon, Hughes, Bashforth, Sibley, and Rutherford, Law and Lowe, are well adapted for the purpose, assuming the surface line of the cross sections to be level, or to be reduced to that position; but none of these tables will properly apply to sectional areas, which is the most important part of their use, excepting Bashforth's; *but his method of using them is erroneous, the error approximating to 50 per cent, as a maximum.* I would therefore recommend for

this purpose, the *General Earthwork Table,** in conjunction with *Two Auxiliary Tables*, on the same sheet in *Baker's Engineering*, as being applicable to all varieties of ratio of slopes and widths of formation level in common use; and with the help of *Barlow's Tables of Square Roots*, these tables will apply to sectional areas, with all the mathematical accuracy that can be attained, with very little more calculation than adding the contents between every two cross sections, as given by the General Table. —The contents in the General Table are calculated to the nearest unit, as are also those in the Auxiliary Table, No. 2, which is for the decimals of feet in the depths. The Auxiliary Table, No. 1, shows the depths of the meeting of the side slopes below the formation level, with the number of cubic yards to be subtracted from the contents of the General Table for each chain in length, for eight of the most common varieties of ratio of slope.

NOTE 1. These Tables, with very little additional calculation, may be extended to every variety of formation level and ratio of slopes that can occur, and even to cases where that ratio differs in the two sides of the same cutting, as shall be shewn in the following Problems.

NOTE. 2. The investigations of the method of forming these tables and using them are given in *Baker's Railway Engineering*, also further investigations are given at the end of the following Problems, respecting *Mr. Bashforth's Erroneous Methods of Calculating Earthwork*.

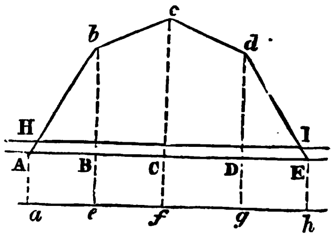
The following diagrams and explanations will further illustrate the method of taking the dimensions of railway cuttings, preparatory to using the above named tables.



Let A B D C *c a b d*, be a railway cutting, of which A B D C, *a b d c* are the cross sections, A B=*a b*= width of formation level, M M', *m m'* the middle depths of the two cross-sections; the side-slopes A C, B D, *a c*, *b d*, when prolonged two and two, will intersect at N and *n*, at which points the prolongations of M M', *m m'* will also meet, thus constituting a prism A B N *n a b*, the content of which is to be deducted from the whole content, given by the General table, by means of the table No 1.; in which the depth M'N=*m'n* is also given, as already stated, to several varieties of slope and bottom width.

* The numbers for the side slopes, forming the alternate lines in *Bidder's Table*, will supply the place of the General Table, and the formula Prob. III., page 199 gives the cubic yards to be deducted for each chain in length, the quantities for the decimals in the depths, as shown by Table No. 2, may be omitted by taking the nearest whole numbers in the depths.

To place this subject in a more practical point of view, let the annexed figure represent a longitudinal and vertical section of a cutting, passing through the middle A E of the formation level. H I, the line of the rails, and *a h*, the line in which the slopes if prolonged, would meet. It will be seen that the cutting A *b c d* E commences and runs out on the formation level A E, and that the depth A *a* = B *e* = C *f* = &c. is to be added to the several depths B *b* C *c*, D *d* of the cutting, the first and last depth at A and E being each = 0; or, what amounts to the same thing



the several depths must be measured from the line *a h*: thus, A *a*, *b e*, *c f*, &c. are the depths to be used. And since the depth A *a* is given in Table No. 1, for all the most common cases, or it may be readily found by calculation for all cases, as shall hereafter be shown, the line corresponding to *a h* must, therefore, be ruled on the railway-section, at the proper distance below A E, from which the several depths must be measured; or the vertical scale may be marked with Indian ink (which may be readily rubbed off) at the same distance, and this mark may then be applied to the formation level A E, for the purpose of measuring the several depths.—In the case of an embankment, the line for the several depths must be placed at a like distance above the formation level.

PROBLEM I.

The several depths of a railway cutting to the meeting of the side slopes, its width of formation level, and the ratio of the slopes being given, to find the content of the cutting in cubic yards, from the Tables referred to, the distances of the depths being one chain each.

RULE.—Take the several quantities corresponding to every two succeeding depths of a cutting or embankment, measured to the meeting of the side slopes, at the distance of 1 chain each from the General Table in *Baker's Railway Engineering* and multiply their sum by the ratio of the slopes; from the product subtract the cubic yards corresponding to the given bottom width and ratio of slopes from Table No. 1., multiplied by the whole length of the cutting, and the remainder will be the content of the cutting in cubic yards.

But when the distances of the depths are greater or less than 1 chain, the quantities of the General Table must be multiplied by their respective distances.—And, when the distances are given in feet, the quantities must be multiplied by those distances, and the final result divided by 66 for the content in cubic yards, as in the following examples.

EXAMPLES.

1. Let the depth of the railway cutting or embankment to the meeting of the side slopes, at the end of every chain, be as in the following table, the bottom width 30 feet, and the ratio of the slopes as 2 to 1; required the content in cubic yards.

Dist. in chains.	Depths in feet.	Qnts. per G. Table.
0	10	
1·00	33	1238
2·00	39	3175
3·00	35	3350
4·00	10	1365
For slope 1 to 1 .		9128
		2
——— 2 to 1.		18256
Subtract 275 × 4		= 1100
Content in cubic yds.		=17156

NOTE. In the annexed table the quantity 1238, corresponds to the depths 10 and 33 feet, in the General Table; the quantity 3175 to the depths 33 and 39, and so on for the succeeding depths. By the Auxiliary Table No. 1, it will be seen, that the depth to be added below the formation level, for the given width and ratio of slopes, is 7·50 = $7\frac{1}{2}$ feet, therefore, the cutting begins and ends with a depth of $10 - 7\frac{1}{2} = 2\frac{1}{2}$ feet. The corresponding number of cubic yards, to be deducted for each chain in length, is multiplied by 4 chains, the whole length of the cutting, thus giving the whole quantity to be deducted, the remainder being the true content in cubic yards of the cutting.

2. The several depths of a railway cutting to the meeting of the side slopes are as in the annexed table, the bottom width being 30 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content of the cutting.

Dist. in chains.	Depths in feet.	Products for Dist. greater than 1 chain.	Total quantities.
0	10		
1·00	16	.	420
2 00	20	. . . . .	795
4·00	25	1243 × 2	2486
5·00	32	. . . . .	1996
7·00	39	3091 × 2	6182
8·00	45	. . . . .	4319
10·00	50	5520 × 2	11040
12·00	40	4971 × 2	9942
13·00	30	. . . . .	3015
14·46	10	1059 × 1·46	1546
For side slopes 1 to 1 . .			41741
——— $\frac{1}{2}$ to 1 . . .			20870
——— $1\frac{1}{2}$ to 1 . .			62611
366·67 × 14·46 =			5302
Content in cubic yards			57309

NOTE. When any of the distances between two succeeding depths is greater or less than 1 chain, the corresponding quantity from the General Table must be multiplied by that particular distance; as the distances between the depths 20 and 25, and between 32 and 39, &c. the distances being each 2 chains. The last distance, viz., that between 30 and 10, is 1·46; in this case 2 figures must be cut off for decimals, after multiplying.

3. Let the depths of a railway cutting to the meeting of the side slopes, and their distances in feet be as in the annexed table, the bottom width 30 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the content in cubic yards.

NOTE. When the distances are in feet the quantities from General Table must be respectively multiplied by their distances, the quantity from Table No. 1 by the whole distance, and the result divided by 66, the feet in 1 chain, for the content in cubic yards, as in the annexed operation. See *Baker's Railway Engineering.*

Dist. in feet.	Depths in feet.	Quantities multipled by length.	Total quantities.
0	37		
90	50	4660 × 90	419400
178	61	7554 × 88	664752
278	39	6210 × 100	621000
For slopes 1 to 1			1705152
——— ½ to 1			852576
——— $1\frac{1}{2}$ to 1			2557728
		366·67 × 278 =	101934
		66)	2455794
Content in cubic yards . . .			37209

PROBLEM II.

CASE I.—*The areas of two cross sections of a railway cutting to the intersection of the side slopes, its length in chains, bottom width, and ratio of the slopes are given; required the content of the cutting in cubic yards.*

RULE.—With the square roots of the given areas, as depths, find the content from the General Table, as in the last Problem, from which subtract the quantity answering to the given width, and the ratio of sides slopes from Table No. 1, and the remainder, being multiplied by the length, will be the content required.

NOTE. If the length be given in feet, proceed as in Example 3, last Problem.

EXAMPLES.

1. Let the two sectional areas of a cutting be 4761 and 1296 square feet, the bottom width 36 feet, the length 3·25 chains, and the ratio of the side slopes 2 to 1; required the content in cubic yards.

$$\left.\begin{array}{l}\sqrt{4761} = 69\\ \sqrt{1296} = 36\end{array}\right\} \text{content per General Table...}\ 6959$$

For bottom width 36 and slopes 2 to 1 per Table No. 1. 396

Content of 1 chain in length 6563

$3\frac{1}{4}$

19689

1641

Content for 3·25 chains.............................. 21330 cubic yds.

CASE II.—*To find the content when the depths are given in feet and decimals of feet.*

RULE.—Let a and b be the feet in any two succeeding depths, and α and β their respective decimal parts; find the quantity answering to a and b from General Table, as in the former cases; then,

$2a + \beta$, rejecting the last figure, and α will shew the number to be added in Auxiliary Table No. 2, and

$2b + \alpha$, rejecting the last figure, and β will shew the number to be added in the same Table.

After which deduct for the quantity below the formation level as before.

EXAMPLE.

Let the sectional areas be 1406 and 2560 square feet, the bottom width 36 feet, length 4 chains, and ratio of slopes $1\frac{1}{2}$ to 1; required the content in cubic yards.

Here $\sqrt{1406} = 37{\cdot}5$ and $\sqrt{2540} = 50{\cdot}4$. Put $37 = a$, $50 = b$, $\cdot 5 = \alpha$, $\cdot 4 = \beta$; then the depths a and b, per General

Table, give	4660
$2a + b = 124$, or 12, (by rejecting last figure) and $\beta = \cdot 4$, per Table No. 2, give	39
$2b + a = 137$, or 14 nearly, (by rejecting last figure) and $\alpha = \cdot 5$ give	57
	4756
For bottom width 36 and ratio of slopes $1\frac{1}{2}$ to 1, deduct	528
Content for 1 chain in length	4228
	4
Ditto for 4 chains in length	16912 cubic yds.

CASE III.—*In measuring contract work, where great accuracy is required, the $\frac{1}{100}$ths of a foot, or second decimals, must be used in the calculation, by taking for them $\frac{1}{10}$th of their respective quantities in Table No. 2.*

EXAMPLE.

Dist. in chains.	Areas in sq. feet.
0	2727
2·00	3136
6·00	4221
9·60	4100
14·00	5142
16·00	3759
18·00	2161

The areas of seven cross sections of a railway cutting to the meeting of the side slopes, and their distances are as in the annexed table; the bottom width is 30 feet, and the ratio of the slopes $1\frac{1}{2}$ to 1; required the cubic yards in the cutting.

Ans. The content, per General Table, and Table No. 2, is 172318 cubic yards, from which the quantity corresponding to the

given bottom width and ratio of slopes × by the whole length, viz. $275 \times 18 = 4950$ cubic yards, must be deducted, which leaves 167568 cubic yards, the content required.

NOTE 1.—When the distances of the sectional areas are given in feet, the quantities of the General Table must be multiplied by their respective distances, and the final result divided by 66, as in Example 3, Prob. I.

NOTE 2.—When the surface lines of the sectional areas are either level or are readily reducible to that position, the decimals, if any, in depths must be taken into the calculation, as in Cases II. and III.

PROBLEM III.

To adapt the General Table to such widths of the formation level and ratios of slope as are not found in Table No. 1.

Put $\omega = \frac{1}{2}$ width of formation level, and $r : 1$ the ratio of slope.

Then $\frac{\omega}{r}$ = feet to be added to the depth of cutting below formation level.

And $\frac{22\,\omega^2}{9\,r}$ = cubic yards to be subtracted for each chain in length.

EXAMPLE.

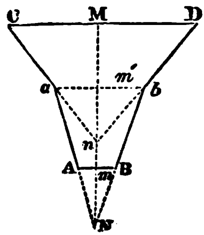
Let the width of formation level be 26 feet, and the ratio of slopes $1\frac{1}{4}$ to 1; then $\omega \div r = 13 \div 1\frac{1}{4} = 10{\cdot}4$ ft. = distance below formation level to meeting of slopes.

And $22 \times 13^2 \div 9 \times 1\frac{1}{4} = 330{\cdot}5$ cubic yards to be deducted for each chain in length from the contents of the General Table.

PROBLEM IV.

To find the content of a cutting, when each of the sides have two different ratios of slope.

RULE.—When the cutting ABCD has two different slopes, as Aa or Bb and aC or bD; it must be divided into two parts by the line $a\,b$, and the quantities of the parts ABba, abDC of the cutting must be found separately by Prob. I. If the surface line CD be sloping or curved, Prob. II. will also be required; and, if the depths $m'\,n$, mN and their corresponding quantities of cubic yards are not found in Table No. 1, they must be found by Prob. III., the sum of the contents of the two parts being the required content of the cutting.



NOTE. Cuttings of this kind are often advantageously adopted, where their upper parts are of loose or springy earth, and their lower parts strong clay or rocky.

PROBLEM V.

To find the content of a cutting when the ratio of the slopes of the two sides are different.

CASE I.—*When the surface is level.*

RULE.—Find the central depths to the meeting of each of the side slopes, and take their corresponding contents by Prob. I. for the whole length of the cutting, in the same manner as if it were for two cuttings, and from the sum, subtract the sum of the cubic yards corresponding to the given bottom width and ratios of slope multiplied by the whole length of the cutting, and half the sum will be the content required.

CASE II.—*When sectional areas are given.*

RULE.—Find the contents corresponding to the sectional areas to the meeting of the side slopes by Prob. II., and from their sum deduct $\frac{44\,\omega^2\,l}{9(r + r')}$ cubic yards for the content. In this formula $\omega = \frac{1}{2}$ bottom width, l = whole length of cutting, and r and r' the first terms of the ratios of the side slopes.

NOTE. Cuttings with slopes of this kind are frequently adopted in practice, where the ground is springy on one side of them, the greater ratio of slope being on the springy side.

PROBLEM VI.

To find the quantity of the cutting of a tunnel.

RULE.—Multiply continually together the width, mean height and length; divide the product by 9, if the length be given in yards, but, if the length be given in chains, multiply the product by 22 and divide by 9, the width and height, in both cases, being given in feet.

NOTE. Examples are not given in the three last Problems, the methods of solving which being sufficiently obvious from the Rules and the first three Problems.

CHAPTER VI.

PROBLEMS AND FORMULÆ OF UTILITY IN LAND AND ENGINEERING SURVEYING.

The areas of triangles, &c., may be expeditiously found by the following Problems, without mapping them for the purpose of finding perpendiculars, &c.

PROBLEM I.

To find the area of a triangle, when two of its sides and their included angle are given.

Let a and b be the two sides of the triangle, α their included angle, and A the area; then

$$A = \tfrac{1}{2}\, a\, b \text{ sine } \alpha$$

This Problem may be solved by logarithms, by which the multiplication of large numbers is avoided, and by which the area is found in acres and decimals; thus

$$\text{log. } A = \text{log. } a + \text{log. } b + \text{log. sine } \alpha - 15{\cdot}30103.$$

EXAMPLE.

The two sides of a triangle are 1920 and 1152 links, and their included angle 53° 8′; required the area.

log. 1920	3·28330
log. 1152	3·06145
log. sine 53° 8′	9·90311
	16·24786
	15·30103
log. A = 8·848	0·94683

or, A = 8*a*. 3*r*. 15*p*., the area required.

PROBLEM II.

To find the area of a triangle, when two of its angles and their included side are given.

Let α and β be the two angles, and a their included side; then

$$A = \frac{a^2 \text{ sine } \alpha \text{ sine } \beta}{2 \text{ sine}(\alpha + \beta)}.$$

By logarithms for the area in acres.

$$\log. A = 2 \log. a + \log. \text{sine } \alpha + \log. \text{sine } \beta - \log. \text{sine } (\alpha+\beta) - 15{\cdot}30103$$

EXAMPLE.

A side of a triangle is 2000 links, and its adjacent angles 60° 14′ and 59° 46′; required the area by logarithms.

Ans. 17*a.* 1*r.* 12*p*

PROBLEM III.

To find the area of a triangle when the three sides are given.

Let a, b and c be the sides, and s their half sum, then

$$A = \sqrt{s(s-a)(s-b)(s-c)}.$$

By logarithms, for the areas as above,

$$\log. A = \log. s + \log. (s-a) + \log. (s-b) + \log. (s-c) \div 2 - 5.$$

EXAMPLE.

The three sides of a triangle are 3050, 2520, and 2040 links, required the area by logarithms. *Ans.* 25*a.* 2*r.* 4*p.*

PROBLEM IV.

To find the area of a trapezium, when its four sides are given, two of its opposite angles being together = 180°.

Let a, b, c, and d be the four sides, and s = half their sum; then, $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$.

The method of solving this Problem by logarithms is sufficiently obvious from Problem III..

PROBLEM V.

To find the area of a trapezium when its two diagonals and the angle of their intersection are given.

Let D and Δ be the two diagonals, and α the angle of their intersection; then

$$A = \frac{D\, \Delta \text{ sine } \alpha}{2}.$$

The method of solving this Problem by logarithms is the same as that given in Problem I.

NOTE. The investigations of the Formulæ used in these Problems are given in various works on Analytical Trigonometry.

PROBLEM VI.

A B C *is a triangle, in which the base* A B *and a point* D *therein are given,* D C *is a quality line, making a given angle with* A B; *it is required to determine* C D *so that the triangle* A B C *may contain land of a given value.*

Put $AD = a$, $DB = b$, $CD = x$, l = square links in an acre, sine $\angle D = \sigma$, the values of the land adjacent to a and b

respectively, m and n per acre, and V the given value; then the areas of the triangles A C D, B C D are respectively $\frac{a\sigma x}{2l}$ and $\frac{b\sigma x}{2l}$ and their values are $\frac{a\sigma m x}{2l}$ and $\frac{b\sigma n x}{2l}$ whence $\frac{a\sigma m x}{2l}+\frac{b\sigma n x}{2l}=\mathrm{V}$, and

$$x=\frac{2\,\mathrm{V}l}{\sigma(am+bn)}=\mathrm{C\,D};$$

whence the position of the point C becomes known.

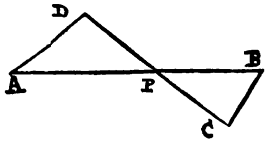
Cor. When C D is perpendicular to A B, $\sigma=1$, whence the above formula becomes

$$x=\frac{2\,\mathrm{V}\,l}{am+bn}=\mathrm{C\,D}.$$

Note. This Problem would be available in solving Case IV, Problem V, page 121; however, it is there proposed to be solved by assumed or "Guess lines," conformable to the old practice of surveyors.

Problem VII.

In making an extensive survey, the fundamental lines A B, B C, C D, D A were measured, and the distance C P noted; but the distance A P was accidentally omitted. It is required to lay down the lines independent of this distance; and give a solution when A D = D P.



Put $\mathrm{AB}=a$, $\mathrm{BC}=b$, $\mathrm{CP}=c$, $\mathrm{PD}=d$, $\mathrm{DA}=e$, and $\mathrm{AP}=x$; then $\mathrm{PB}=a-x$; and by trigonometry $\cos.\angle\mathrm{APD}=\cos.\angle\mathrm{BPC}=\frac{x^2+d^2-e^2}{2dx}=\frac{(a-x)^2+c^2-b^2}{2c(a-x)}$; whence

$$x^3-\frac{a(c+2d)}{c+d}x^2+\frac{d(a^2-b^2+c^2)+c(d^2-e^2)}{c+d}x-\frac{ac(d^2-e^2)}{c+d}=0,$$

a cubic equation, from which the value of x may be found, which determines the distance A P.

When A D = D P, that is, when $d=e$, the above equation becomes

$$x^2-\frac{a(c-2d)}{c+d}x+\frac{d(a^2-b^2+c^2)}{c+d}=0,$$

a quadratic equation, from which the value of $x=\mathrm{AP}$ may be readily found.

Investigation of formulæ.—As the formulæ for dividing land, at pages 119 and 121, Cases II. and III., Problem V.,

are given without investigation, to obviate all doubts of their accuracy, the processes of deducing them are here given.

(1.) By referring to the definition of the symbols, and to the diagram at page 119, Case II., it will be seen that the right angled triangles W S s, W U u, W F b, are similar, therefore,

$$s : x :: a : \frac{a\,x}{s} = \text{S}\,s,$$

$$s : x :: a + b : \frac{(a + b)\,x}{s} = \text{U}\,u; \text{ hence}$$

the area in acres of W S $s = \frac{1}{2}$ W S $\cdot$ S$s = \dfrac{a^2 x}{2\,s\,l}$,

———————— of Ss U $u = \frac{1}{2}$(Ss + Uu) S U $= \dfrac{(2\,a+b)\,b\,x}{2\,s\,l}$,

———————— of U u F $b = \frac{1}{2}$(Uu + Fb)U F $= \dfrac{(2\,a+2\,b+c)\,c\,x}{2\,s\,l}$

Then these areas, being multiplied by their respective values per acre, and their sum equated to the required value V′, give

$$\frac{a^2 m\,x}{2\,s\,l} + \frac{(2\,a + b)\,b\,n\,x}{2\,s\,l} + \frac{(2\,a + 2\,b + c)\,c\,o\,x}{2\,l\,s} = \text{V}',$$

whence $x = \dfrac{2\,l\,s\,\text{V}'}{a^2 m + (2a+b)bn + (2a+2b+c)co} = \text{F}\,b.$ Q. E. I.

Cor. I. If there be only two qualities of land to be thus laid out, c must be made to vanish in the above formula, whence it becomes

$$x = \frac{2\,l\,s\,\text{V}'}{a^2 m + (2a+b)bn} = \text{F}\,b.$$

Cor. II. If there be four different qualities of land, let the breadth of the additional quality be d, and its value p; then the formula becomes

$$x = \frac{2\,l\,s\,\text{V}'}{a^2 m + (2\,a+b)\,b\,n + (2\,a+2\,b+c)co + (2\,a+2\,b+2\,c + d)\,d\,p} = \text{F}\,b$$

the law of extension being sufficiently clear.

(2.) By referring to the diagram at page 120, Case III., Problem V., and to the symbols on the same and following page, it will be readily perceived, that by drawing a perpendicular from a on N B, (which is not shewn in the diagram) that $a\,q = a - 2\,x\,\alpha$, whence the area in acres of the trapezoid N $a\,q\,p = \frac{1}{2}$(N $p - a\,q$) $p\,q = \dfrac{(a - \alpha\,x)\,x}{l}$, in a similar way the area of B $b\,s\,r$ is found $= \dfrac{(c - \beta\,x)x}{l}$, the area of the rect-

angle $p\,q\,r\,s$ being obviously $= \frac{b\,x}{l}$. By multiplying these three areas by their respective values, and putting their sum equal to the required value V′, their results

$$\frac{(a - \alpha x)\,m\,x + (c - \beta x)\,o\,x + b\,n\,x}{l} = V'; \text{ whence}$$

$(\alpha m + \beta o)\,x^2 - (a\,m + b\,n + c\,o)\,x + l\,V' = 0$; and, by solving this quadratic for the reciprocal of x, there results

$$x = \frac{2\,l\,V'}{am + bn + co + \sqrt{(am + bn + co)^2 - 4(\alpha m + \beta o)\,l\,V'}} = pq \text{ or } rs;$$

NOTE The method of adapting this formula to a greater or less number of qualities of land is sufficiently clear, from the different modifications of the preceding formula in Cor. I. and II.

PROBLEMS, ORIGINAL AND SELECT,

FOR THE EXERCISE OF STUDENTS.

I. The side of an equilateral triangle is $= a$, and its area $=$ A; prove that

$$A = \tfrac{1}{4}\,a^2\,\sqrt{3}.$$

II. The base of an isosceles triangle is $= a$, one of its equal sides $= b$, and its area A; prove that

$$A = \tfrac{1}{4}\,a\,\sqrt{(2\,b + a)\,(2\,b - a)}.$$

III. In a triangle are given the perpendicular $= p$, the angles opposite the perpendicular $= \alpha$, and β, and consequently the third angle $= \gamma$; prove that

$$A = \frac{p^2 \text{ sine } \gamma}{2 \text{ sine } \alpha \text{ sine } \beta}.$$

IV. ABCD is a trapezium, in which the angles at A and C are equal, and AB $= a$, BC $= b$, CD $= c$, DA $= d$, the half sum of all the sides $= s$, and the area $=$ A; then prove that

$$A = \frac{a\,d + b\,c}{a\,d - b\,c}\sqrt{s(s - a - c)\,(s - b - c)\,(s - c - d)}.$$

V. In a trapezium A B C D, are given two opposite sides A B $= a$, C D $= b$, the angles at A, B, and C respectively $= \alpha$, β, and γ, and consequently the fourth angle D $= \delta$; then prove that

$$A = \frac{a^2 \text{ sine } \alpha \text{ sine } \beta - b^2 \text{ sine } \gamma \text{ sine } \delta}{2 \text{ sine } (\alpha + \beta)}.$$

VI. In a trapezium A B C D, are given the three sides

$CD = a$, $BA = b$, $AD = c$, and the angles at A and B respectively $= \alpha$ and β; then prove that

$$A = \tfrac{1}{2}\left\{ a\,b \text{ sine } \beta + b\,c \text{ sine } \alpha - a\,c \text{ sine } (\alpha + \beta) \right\}$$

VII. In a pleasure ground A B C D, known to be rectangular, only the following dimensions could be taken, on account of obstructions from buildings, shrubberies, and ponds, viz., the distance A E to the point where the perpendicular B E falls on the diagonal A C, and the prolongation E F of B E till it meet the side C D; it is required to find the area of the pleasure ground, when $AE = 32$, and $EF = 4$ chains.

Ans. 64 acres.

VIII. A large building is known to be of a square form, but no one of its sides could be measured on account of obstructions from other buildings; however, from a point P three streets diverge directly to its three nearest angles A, B, and C. Now $PA = 60$, $PB = 40$ and $PC = 70$ yards; required the side of the building.

IX. Within a rectangular garden, the length of which is 4 and its breadth 3 chains, there is a piece of water in the form of a trapezium, the opposite angles of which are in a direct line with those of the garden, the distances of these opposite angles, taken in succession, are 20, 25, 40 and 45 yards; required the area of the water. *Ans.* 960 square yards.

X. Given the three perpendiculars of a triangle, from the angular points to their opposite sides, 10, 11, and 12 chains, to find the area of the triangle. *Ans.* 7*a.* 0*r.* $4\frac{1}{2}$ *p.*

XI. Three objects, A, B, C, are observed at a point D, exterior to them; the distances of the objects are known to be as follows, $AB = 8$, $BC = 12$, and $AC = 7\frac{1}{8}$ miles: the angles A D C, A B D are respectively 25° and 19°; required the distances A D, B D, and C D.

XII. A gentleman intends to make an elliptical garden, the principal axes of which are to be as 3 to 1, and the fence of which is to include three trees, one at the end of the transverse axis, the second 6 poles from it, and the third the same distance from the second, the three trees forming a right angle at the second; required the axes and area of the garden. (*Gentleman's Mathematical Companion, Quest.* 449.)

Ans. Minor axis 55·87 *poles, area* 73·55 *sq. poles.*

XIII. In an elliptical enclosure of one acre the principal axes are as 5 to 4; required the length of a chord, which

fastened to the end of the longer axis, will allow a horse to graze half an acre. (*G. M. Comp. Quest.* 345.)

Ans. 48·45473 *yards.*

XIV. Given the diagonals and two opposite sides of a trapezium to construct it, when the area is a maximum or a minimum. (*Prize Quest. G. M. Comp. for* 1809.)

XV. In a trapezium ABDC are given all the sides $AB = a$, $BD = b$, $DC = c$, $CA = d$, and the diagonal $BC = e$ to find the diagonal AD, without constructing the figure. (*B. Gompertz's Principles of Imaginary Quantities, page* 29. *Prob. V.*)

$$\textit{Ans.}\ AD = \sqrt{a^2 + b^2 - ab \times \frac{gh - \sqrt{4 - h^2} \times \sqrt{4 - g^2}}{2}}$$

$$\text{in which } g = \frac{a^2 + e^2 - d^2}{a\,e}, \text{ and } h = \frac{b^2 + e^2 - c^2}{b\,e}.$$

XVI. There are given the four sides of a trapezium, and a line joining given points in two of its opposite sides, from which it is required to construct the figure.

XVII. The five sides a, b, c, d, and e of an irregular field are given, in which the angles between a and b, b and c, c and d, are equal but not given; from these *data* it is required to lay down the field.

NOTE. The solutions of the two last Problems are obtainable by cubic equations. The former was proposed by the author, in the *Gentleman's Diary* for 1838, it having occurred in his practice of town surveying for railway purposes. The latter was proposed by *B. Gompertz, Esq., F.R.S., &c.*, in the *Gentleman's Mathematical Companion;* to which he gave a solution in a concise, novel, and ingenious manner by his *Principles of Imaginary Quantities:* other solutions by the ordinary methods were also given to the same Problem.

XVIII. A gentleman has an elliptical garden, the principal axes of which are 50 and 40 yards, enclosed by a brick-wall 13 feet high. He ordered his gardener to place his seat at equal distances from the centre, one of the foci and the boundary of the garden, and around the seat to make a gravel walk of equal breadth taking up $\frac{1}{20}$ of the area of the garden, and to be of such a nature, that, while the gentleman is seated, and the gardener moving along the middle of the walk, the gentleman's eye, the gardener's utmost height, and the top of the wall, may be in the same right line; the height of the gentleman's eye, (when seated), and that of the gardener, being 4 and 6 feet respectively. Required the position of the seat, and the nature and breadth of the walk. (*G. M. Comp. for* 1806, *Quest.* 14.)

Ans. The equal distances of the seat from the centre, focus,

and curve = 10·7812 *yards; the walk is elliptical, its axes being* $11\frac{1}{9}$ *and* $9\frac{1}{9}$ *yards, its breadth* $2\frac{1}{2}$ *yards, and the position of the walk with respect to the seat being the same as the fence of the garden.*

NOTE. The solutions to all the Questions, taken from the works referred to, may be seen in those works.

(DIVISION OF LAND.)

XIX. In a triangle ABC, AB = a, BC = b, CA = c, and from D, a given point in AB, the distance DB = d, DE a line meeting BC and dividing the triangle, so that $\triangle$ ABC : $\triangle$ BDE :: m : n; then prove that

$$DE = \frac{a\,b\,n}{d\,m}.$$

But if it be found from the calculation that BE is greater than BC, and that the divisional line will meet AC in some point F; then prove that

$$AF = \frac{a\,c\,(m - n)}{m\,(a - d)}.$$

XX. ABCD is a given trapezium, and T a given point in AD, from which a line TE is drawn to meet BC in E, so that trapezium ABCD : trapezium DCET :: m : n. Now, since the trapezium ABCD is given, if the sides AD, BC be prolonged till they meet in Z, the point Z will also be given; therefore, put AZ = a, BZ = b, DZ = c, CZ = d, and TZ = f; then prove that

$$EZ = \frac{n\,(a\,b - c\,d)}{m\,f} + \frac{c\,d}{f}.$$

XXI. It is required to divide a given trapezium into four equal parts, by two right lines perpendicular to one another.

NOTE. This Problem was proposed, the author believes, in one of the *Diaries*, many years ago, where its solution may be seen.

(RAILWAY ENGINEERING.)

XXII. Let δ be distance between the tangent points of a serpentine curve, consisting of two circular arcs of equal radii r, α and β the angles that δ makes with the tangents, and σ = arc to $\frac{1}{2}$ (cos. α + cos. β); then prove that

$$r = \frac{\delta}{\text{sine } \alpha + \text{sine } \beta + 2 \text{ sine } \sigma}.$$

XXIII. Find the distances of the tangent points to the point of contrary flexure of the curve, in the last Problem, and the

angles that these distances make with the tangents to the curve, and give the requisite formula.

XXIV. The widths of a laterally sloping railway cutting from the centre of the line are expressed generally by the following formula.

$$\omega = \frac{b\,s}{l \pm r\,h},$$

the positive sign being used for the width measure down the slope, and the negative one for that up the slope, in which formula $b = \frac{1}{2}$ width of the cutting, assuming its surface to be level, h = differences of level readings at the distances s and l on the slope and level, and r the ratio of the slopes. See figure to Prob. II., Chap. III.

XXV. Let a and b be the depth of a railway cutting to the intersection of the slopes, l = length of the cutting, $\omega = \frac{1}{2}$ bottom width, all in feet, and r = ratio of the slopes, the surface of the cutting being assumed to be horizontal; then prove that the content of the cutting in cubic yards is

$$\frac{l\,r}{81}\left(a^2 + a\,b + b^2 - \frac{3\,\omega^2}{r^2}\right).$$

XXVI. When a and b are the depths of a horizontal cutting to the formation level, and the other dimensions the same as in the last Problem; then prove that the content of the cutting in cubic yards is

$$\frac{l\,r}{81}\left\{a^2 + a\,b + b^2 - \frac{3\,\omega}{r}(a + b)\right\}.$$

XXVII. Let A and B be the areas of the cross sections of a cutting to the intersection of the slopes, d, the area of the end of the prism below the formation level, (see fig. p. 194) and l the length of the cutting; then prove that the content in cubic yards is

$$\frac{l}{81}(\mathrm{A} + \mathrm{B} + \sqrt{\mathrm{A\,B}} - 3\,d).$$

XXVIII. Let the dimensions be as in Problem XXV then prove that the error in defect of the method of finding the content of a cutting by using the mean depths is

$$\frac{l\,r}{324}(a - b)^2.$$

XXIX. Let the dimensions be as in Problem XXVII.;

then prove that the error in excess of the method of finding the content of a cutting by using mean areas is

$$\frac{l}{162}(\sqrt{A} - \sqrt{B})^2.$$

XXX. Prove the formula at p. 202, for finding the error per cent. of Mr. Bashforth's method of finding the contents of railway cuttings from sectional areas.

NOTE. The demonstrations of the nine last Problems are given in *Baker's Railway Engineering*.

CENTRIFUGAL FORCE OF TRAINS IN RAILWAY CURVES.

Since all moving bodies have a tendency to continue their motion in a direct line, from this cause the carriages of a railway train of great velocity are strongly urged towards the outer rail, and would ultimately be driven off the rails, were it not for the flanges of the wheels and the conical inclination of their tire.

Let F = centrifugal force thus generated, W = weight of the train, V = its velocity, R = radius of the curve, in which the train moves, and g the force of gravity at the earth's surface; then by Dynamics

$$F = \frac{W\,V^2}{g\,R}.$$

EXAMPLE.

1. When R = $\frac{1}{2}$ a mile = 2640 feet, V = velocity = 30 miles per hour = 44 feet per second, and $g = 32\frac{1}{6}$ feet = velocity of a body falling from rest, at the end of a second; then

$$F = \frac{W \times 44^2}{32\frac{1}{6} \times 2640} = \frac{22}{965}\,W = \text{nearly } \frac{1}{44}\,W,$$

that is, the force that urges the train to quit the curve is $\frac{1}{44}$ of its whole weight, in this case.

2. When V = 60 miles per hour = 88 feet per second, and R the same as in Example 1; then

$$F = \frac{W \times 88^2}{32\frac{1}{6} \times 2640} = \text{nearly } \frac{1}{11}\,W,^*$$

* This great amount of centrifugal force, in curves of small radius, would be very much increased by the high velocities, which some are sanguine enough to expect as likely to be attained on railways; since this force varies as

$$\frac{V^2}{R} \text{ or as } V^2$$

that is the force, in this case, is $\frac{1}{11}$ of the weight of the train. Hence it may be perceived how extremely dangerous high velocities are in curves of small radius.

3. When the radius is = 1 mile = 5280 feet, and V the same as in example 2; then

$$F = \tfrac{1}{22} W.$$

This force, except in curves of very small radius, is counteracted by the conical inclination of the tire of the wheels of the engine and its train. The inclination with the lateral play of the flanges of the 2 wheels of about $\frac{1}{2}$ an inch on each side, and the centrifugal force urging the train towards the outer rail, when moving in a curve, increase the diameter of the outer wheel and diminish that of the inner one, which causes the train to roll on a conical surface, thus necessarily producing a centripetal force to counteract the tendency of the train to leave the curve. However, in curves of very small radius, the centripetal force thus generated, does not sufficiently counteract the centrifugal force, a proper super-elevation of the exterior or outer rail being required for this purpose; for determining which *Pambour* has given in his work on *Locomotive Engines*, the following.

FORMULÆ FOR THE SUPER-ELEVATION OF THE EXTERIOR RAIL.

Let V = velocity of the train, R = radius of railway curve, R′ = radius of the curve that the train would describe in consequence of conical shape of the tire of the wheels, and the centrifugal force impelling the train outward, and enlarging the diameter of the outer, and diminish that of the inner wheel, e = gauge of rails, g = force of gravity, and x = super-elevation of outer or exterior rail; then

for the same curve: thus for a velocity of 120 miles per hour, on a curve of $\frac{1}{2}$ a mile radius, we shall have

$$f = \frac{W \times 176^2}{32\frac{1}{6} \times 2640} = \frac{4}{11} W,$$

that is, the centrifugal force is, in this case, more than $\frac{1}{3}$ of the whole weight of the train; while for curves of 1 mile radius, which are very common in railways, $f = \frac{2}{11}$ W, or nearly $\frac{1}{6}$ of the weight of the train. It must, therefore be evident that a velocity of 120 miles per hour, or even one of 90 miles per hour, must be extremely dangerous, especially on an embanked curve, should any accident throw the train off the line, which is often the case with the present velocities. Moreover, the resistance of the air, which varies as V^2, must be considerably augmented by high winds opposed to the direction of a train of these great velocities; while its engine would require a power greatly superior to those now in use.

$$x = \frac{e V^2}{9} \left(\frac{1}{R} - \frac{1}{R'} \right),$$

in which

$$R' = \frac{d e n}{4 \Delta},$$

d being = outer diameter of the wheels, Δ = deviation of the wheels, and $\frac{1}{n}$ = the inclination of the tire.

Concise and, he trusts, clear demonstrations of the above formulæ are given by the author, in his *Railway Engineering.*

By solving these formulæ for some of the usual cases, *Pambour* produces the following.

TABLE OF THE SUPER-ELEVATION TO BE GIVEN TO THE EXTERIOR RAIL IN CURVES.

Designation of the Waggons and the Way.	Radius of the Curve in Feet.	Super-elevation to be given to the Rail in Inches, the Velocity of the motion in Miles per hour being:—		
		10 Miles.	20 Miles.	30 Miles.
Waggon with wheels 3 feet in diameter. Gauge of way 4·7 feet. Play of the waggons on the way, 1 inch. Inclination of the tire of the wheels 1 in 7.	250	1·14	5·60	12·99
	500	0·57	2·83	6·56
	1000	0·29	1·43	3·30
	2000	0·15	0·71	1·65
	3000	0·10	0·47	1·10
	4000	0·07	0·36	0·83
	5000	0·06	0·28	0·66

The correctness of the above results is pretty generally conceded. It must, however, be considered, that it is extremely difficult, if not impossible, to realize in practice, the precise conditions and proportions determined by these important formulæ; as accidental depressions and enlargements of guage of part of the rails, as well as many other matters that cannot be subjected to calculation, will unavoidably derange these results.

The reader, who wishes for further information on these subjects, may consult *Tredgold on the Steam Engine; Hann's Treatise on the Steam Engine;* and *Baker's Statics and Dynamics.*

APPENDIX.

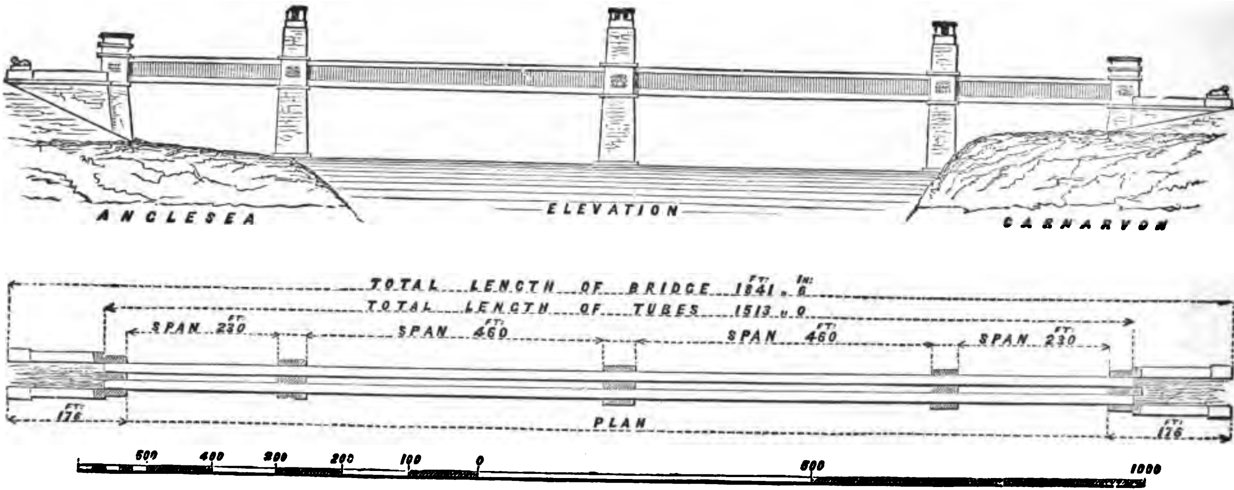
RAILWAY VIADUCTS OR BRIDGES.

There are few parts of a railway which strike the eye so much as the viaducts and bridges, some of which form the masterpieces of our railway engineers. The dimensions of the chief specimens are here given.

TUBULAR AND OTHER IRON GIRDER BRIDGES.

The Britannia tubular bridge.—This structure, combining unparallelled magnitude, strength, and novelty, forms one of the viaducts of the *Chester and Holyhead Railway.* It crosses the Menai Straits, uniting the shores of the mainland of Wales and the Isle of Anglesea. It consists of two rectangular tubes, each 1513 feet in length, or $\frac{2}{7}$ of a mile, 26 feet in average depth, and 14 feet 8 inches in width, the internal depth and width being respectively reduced by the construction to 22 and 14 feet. Each tube has four spans, and consequently three piers or towers, exclusive of the abutments. The two middle spans, in each tube, are each 460 feet, and the two end spans each 230 feet, exclusive of the widths of the towers, which support the tubes at a height of 102 feet above high water mark, the whole height of the middle tower being 200 feet above high water mark, or 230 feet from its foundation. The parts of the tubes forming the middle spans were 472 feet in length, previous to their being united, and weighed upwards of 1600 tons, and were raised to their present lofty position by hydraulic presses worked by steam engines, thus leaving the navigation of the Menai Straits uninterrupted.

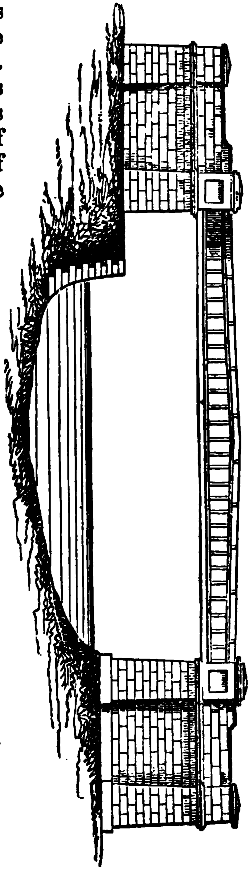
"It is seldom," says Mr. G. D. Dempsey, "that the invention of works of new design and skilful mechanical arrangement is due entirely to one mind, any more than their construction is due to one pair of hands: hence great difficulty arises in assigning to each contributor his fair share of merit in their production. It must, however, be admitted, that to Mr. R. Stephenson alone we are in this instance indebted for the original suggestion; and with this admission, we have endeavoured to avoid any attempt to judge of the precise claims of the two eminent men, whose joint labours have produced the



ANGLESEA
ELEVATION
CARNARVON
TOTAL LENGTH OF BRIDGE FT. 1841 IN. 6
TOTAL LENGTH OF TUBES 1513 0
SPAN FT. 230
SPAN FT. 460
SPAN FT. 460
SPAN FT. 230
FT. 176
PLAN
FT. 176
500
400
300
200
100
0
500
1000

Britannia and Conway Tubular Bridges. That these great works owe their design and construction to their *joint* labours is clearly evident, and, we respectfully submit, amply sufficient to justify the record of the two names of ***Robert Stephenson*** and ***William Fairbairn*** in an honourable and enduring association." The machinery for raising these immense tubes was designed and executed by ***Messrs. Easton and Amos.***

The Conway tubular bridge, in the same line of railway, preceded the Britannia, having been raised in 1848. It consists of one span only of 400 feet, clear width: the height of the tubes above the level of high water is inconsiderable, when compared with the Britannia tubes, being only 18 feet. The bridge thus consists of two tubes only, each weighing 1300 tons. It is erected close beneath the ancient wall of Conway Castle, its abutments being of strong masonry, the designs of which are in harmony with that of the castle.



IRON TUBULAR GIRDER BRIDGES.

The first tubular girder bridge designed and constructed by Mr. Fairbairn, was for the purpose of carrying the Blackburn and Bolton Railway over the Leeds and Liverpool Canal. The span of this bridge is 60 feet, the two lines of rails being carried between three parallel girders.—The cellular work in girders of this kind constitutes their great strength, combined with comparative lightness, the same kind of cellular work being introduced both in the top and bottom of the tubes of the Britannia and Conway Bridges.

The Gainsborough tubular girder bridge is the largest yet constructed of this kind. It forms the viaduct of the Manchester, Sheffield, and Lincolnshire Railway over the Trent; and consists of two spans, each 154 feet wide, with a central pier and abutments of masonry, and two end arches each 40 feet span. This bridge crosses the river obliquely

the abutment and pier are therefore placed at an angle of 50° with the longitudinal direction of the girders, which are two in number, and of uniform depth throughout, between which the double line of railway is carried. This elegant specimen of wrought ironwork was designed and executed by Mr. W. Fair bairn, whose name is so honourably connected with the Britannia and Conway Tubular Bridges.

BRICK AND STONE VIADUCT.

The brick viaduct at Maidenhead, constructed by Mr. Brunel over the Thames on the Great Western Railway, is one of the finest specimens of that material. It consists of two elliptical arches, each 128 feet span, with a rise of 24¼ feet; the pier between the two arches is 30 feet in width; each arch, at the crown, is 5¼ feet in thickness, which increase gradually towards the abutments. Besides these two splendid arches, there are eight others, each of 28 feet span, four on each bank of the river.

The stone viaduct over the Ouse, near York, on the Great North of England Railway, is another specimen combining great strength and elegance of construction. It consists of three arches, each 66 feet span, the width of the arches is 28½ feet, their thickness at the keystone 3½ feet, which gradually increases towards the springing, and the piers are 10 feet in thickness.

The Vale Royal stone viaduct, over the river Weaver, on the Grand Junction Railway, has five arches, each 68 feet span, and 60 in height; the whole length of the viaduct is 456 feet. In the same railway, over the Mersey and Travell Canal, there is a viaduct of 12 stone arches, the central two are each 75 feet span, and the rest from 40 to 14 feet.

The Quaker's Yard viaduct, over the Taff, in the *Taff Vale Railway*, has six arches; its height above the river is 100 feet, and its length 600 feet. There is another viaduct in the same line, at the junction of the Rhondda and Taff, having an arch 100 feet in span, and 60 in height.

The Ouse Valley viaduct, in the London and Brighton Railway, is 1440 feet in length; it has 37 brick arches of 30 feet span, its height varying from 40 to 96 feet.

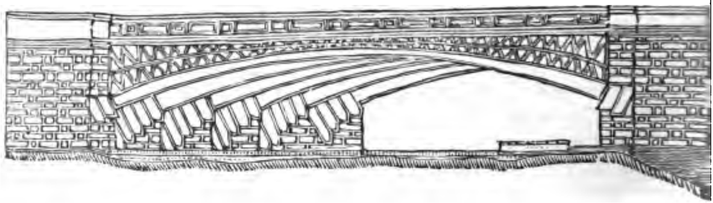
The London and Greenwich Railway, and *the London and South Western Extension* are continuous viaducts; the former consists of more than 1000 brick arches, each 18 feet span, and 24 in height; the arches in the latter are a little wider, having, besides, several elegant oblique iron arches, for

the purpose of crossing Westminster Road and other principal streets. The former is about four miles in length, and the latter nearly two miles.

OBLIQUE OR SKEW BRIDGES.

The Brampton viaduct, over the Gelt, on the Newcastle and Carlisle Railway, crosses the Gelt and a public road, at the height of 80 feet above the bed of the river. It consists of three stone arches of 33 feet in direct span, which are built at an inclination of 45°, thus making their oblique span nearly 50 feet each.

The Fairfield Street bridge, on the Manchester and Bir-



mingham Railway, has an oblique span of 129½ feet, with a rise of 12 feet, and a direct width of 31 feet. It is composed of six ribs of iron, projecting before each other 13 feet, on independent abutments. Open wrought stone parapets crown this viaduct, presenting an agreeable and novel appearance. This bridge was designed by Mr. G. W. Buck, being remarkable for its acute angle, which is only 24½°. The weight of the iron in the six ribs or girders is 540 tons: the remainder of the viaduct consists of brick arches 45 feet in span.

NOTE. Other lines of railway present a great variety of these bridges, both of brick and iron; some of which display great elegance of construction. This kind of bridge, it is said, was first introduced by Mr. Chapman, as canal aqueducts in Ireland.

HIGH LEVEL BRIDGE, NEWCASTLE-UPON-TYNE.

This bridge is of cast iron, and forms the viaduct of the Great North of England Railway across the deep valley of the Tyne. In addition to the elegance and magnitude of this work, it has a common carriage way suspended beneath the railway, thus giving it a decided claim to novelty of construction.

The number of its iron arches are six. Each arch consists of four cast iron ribs, having a span of 125 feet, rising 17¼ in the centre. The ribs are disposed in pairs: one pair on each side of the carriage road. The carriage road is 20¼ feet wide. The foot roads (one between each pair of ribs) 6⅛ feet wide.

Total width of railway 33 feet. Weight of iron in each arch 576 tons; in the whole structure about 5000 tons. Height from high water to level of rails 108½ feet; to suspended carriage road 85 feet; total height from bottom of river 128 feet. Length of bridge from the Castle Garth to Gateshead, inclusive of cast iron approaches, 1337½ feet.

WOODEN BRIDGES.

The Ouseburn viaduct, on the Newcastle and North Shields Railway, has nine arches, of which two at the ends are stone, the other seven are of wood, on stone piers; the three central arches are each 116 feet span, and the two others each 100 feet. The whole length of the viaduct is 756 feet, and its height above the stream in the centre is 180 feet.

The Willington Dean viaduct, on the last mentioned line, consists of seven wooden arches, of which five have a span of 120 feet each, and the two exterior ones 115 feet each. The whole length of the viaduct is 1050 feet, and its height 82 feet.

Wooden viaducts, on a smaller scale, abound in various lines of railway in this kingdom.—Wooden bridges on the *lattice principle* (first suggested by Mr. G. Smart, about 25 years ago, as applicable both to iron and wooden bridges), abound in the United States of America. One of them, over the Susquehannah at Columbia, has 29 spans, each 200 feet wide, the whole viaduct being about 1¼ mile in length. These viaducts can not be anticipated to be of long duration. The *lattice principle* has been considerably improved, and employed in wrought iron bridges by Mr. R. B. Osborne. This principle has been still further extended by Mr. W. C. Harrison, by adding tubular frames to the lattice work above and below, the upper one being bent like an arch. In a design made by him for a viaduct of this kind to carry a railway over the river Ouse, the span is 170 feet, and the rise of the arched tube above the horizontal one is 15 feet. The arched tube to be of wrought iron plates ½ an inch in thickness, and its section to be 4 feet in depth and 3 in width, the horizontal tube to be a little less. For a double line of railway three of these bow frames are required.

No. 1.

Offsets at the end of the first chain from tangent point of railway curves.

Radius of curve in chns.	Offsets in inches and decimals.	Radius of curve in chns.	Offsets in inches and decimals.	Radius of curve in chns.	Offsets in inches and decimals.	Radius of curve in chns.	Offsets in inches and decimals.
5	80·0082	54	7·3340	105	3·7715	186	2·1290
6	66·4649	55	7·2006	106	3·7359	188	2·1064
7	56·8630	56	7·0720	108	3·6667	190	2·0842
8	49·6948	57	6·9480	110	3·6001	192	2·0625
9	44·2326	58	6·8281	112	3·5358	194	2·0412
10	39·7995	59	6·7123	114	3·4738	195	2·0308
11	36·0747	60	6·6005	115	3·4435	196	2·0204
12	33·0575	61	6·4922	116	3·4139	198	2·0000
13	30·5067	62	6·3875	118	3·3560	200	1·9800
14	28·3216	63	6·2862	120	3·3001	205	1·9317
15	25·9002	64	6·1879	122	3·2459	210	1·8857
16	24·7742	65	6·0927	124	3·1935	215	1·8418
17	23·3143	66	6·0004	125	3·1681	220	1·8000
18	22·0170	67	5·9108	126	3·1428	225	1·7600
19	20·8566	68	5·8239	128	3·0937	230	1·7217
20	19·8124	69	5·7395	130	3·0462	235	1·6851
21	18·8679	70	5·6574	132	3·0000	240	1·6500
22	18·0093	71	5·5777	134	2·9552	245	1·6163
23	17·2255	72	5·5003	135	2·9334	250	1·5840
24	16·5072	73	5·4249	136	2·9117	255	1·5529
25	15·8463	74	5·3516	138	2·8645	260	1·5231
26	15·2364	75	5·2802	140	2·8286	265	1·4943
27	14·6717	76	5·2107	142	2·7887	270	1·4667
28	14·1474	77	5·1431	144	2·7500	275	1·4400
29	13·6579	78	5·0772	145	2·7311	280	1·4143
30	13·2037	79	5·0129	146	2·7123	285	1·3995
31	12·7775	80	4·9500	148	2·6756	290	1·3655
32	12·3780	81	4·8891	150	2·6400	300	1·3200
33	12·0027	82	4·8294	152	2·6052	305	1·2983
34	11·6496	83	4·7713	154	2·5714	310	1·2774
35	11·3166	84	4·7145	155	2·5549	320	1·2375
36	11·0021	85	4·6590	156	2·5384	325	1·2184
37	10·7047	86	4·6048	158	2·5063	330	1·2000
38	10·4239	87	4·5519	160	2·4750	335	1·1821
39	10·1555	88	4·5001	162	2·4444	340	1·1647
40	9·9015	89	4·4496	164	2·4146	345	1·1478
41	9·6600	90	4·4001	165	2·4000	350	1·1314
42	9·4303	91	4·3517	166	2·3855	355	1·1155
43	9·2102	92	4·3045	168	2·3571	360	1·1000
44	9·0012	93	4·2582	170	2·3294	365	1·0809
45	8·8005	94	4·2129	172	2·3023	370	1·0703
46	8·6106	95	4·1685	174	2·2758	375	1·0560
47	8·4265	96	4·1251	175	2·2629	380	1·0421
48	8·2511	97	4·0826	176	2·2500	385	1·0285
49	8·0825	98	4·0410	178	2·2248	390	1·0154
50	7·9208	99	4·0001	180	2·2000	395	1·0025
51	7·7655	100	3·9601	182	2·1758	400	·9900
52	7·6161	102	3·8824	184	2·1521		
53	7·4725	104	3·8078	185	2·1406		

No. 2.—Difference between apparent and true level for distances in chains. Correction in decimals of feet.

Dist. in chains.	For curvature.	For refraction.	For curvature and refraction.
3½	·001	·000	·001
4	·002	·000	·002
4½	·002	·000	·002
5	·003	·000	·003
5½	·003	·000	·003
6	·004	·000	·004
6½	·004	·001	·003
7	·005	·001	·004
7½	·006	·001	·005
8	·007	·001	·006
8½	·007	·001	·006
9	·008	·001	·007
9½	·009	·001	·008
10	·010	·001	·009
10½	·011	·002	·009
11	·013	·002	·011
11½	·014	·002	·012
12	·015	·002	·013
12½	·016	·002	·014
13	·018	·002	·019
13½	·019	·003	·016
14	·020	·003	·017
14½	·022	·003	·019
15	·023	·003	·020
15½	·025	·004	·021
16	·027	·004	·023
16½	·028	·004	·024
17	·030	·004	·026
17½	·032	·005	·027
18	·034	·005	·029
18½	·036	·005	·031
19	·038	·005	·033
19½	·040	·006	·034
20	·042	·006	·036
20½	·044	·006	·038
21	·046	·007	·039
21½	·048	·007	·041
22	·050	·007	·043
22½	·053	·007	·046
23	·055	·008	·047
23½	·057	·008	·049
24	·060	·009	·051
24½	·062	·009	·053
25	·065	·009	·056
25½	·068	·010	·058
26	·070	·010	·060
26½	·073	·010	·063
27	·076	·011	·065
27½	·079	·011	·068
28	·082	·012	·070
28½	·085	·012	·073
29	·088	·012	·076
29½	·091	·013	·078
30	·094	·013	·081

No. 3.—Difference between apparent and true level for distances in miles. Correction in feet and decimals.

Dist. in miles.	For curvature.	For refraction.	For curvature and refraction.
¼	·04	·01	·03
½	·17	·02	·15
¾	·37	·05	·32
1	·67	·09	·58
1½	1·50	·21	1·29
2	2·67	·38	2·29
2½	4·17	·60	3·57
3	6·00	·86	5·14
3½	8·17	1·17	7·00
4	10·67	1·52	9·15
4½	13·55	1·93	11·62
5	16·67	2·38	14·29
5½	20·18	2·88	17·30
6	24·01	3·43	20·58
6½	28·18	4·03	24·15
7	32·68	4·67	28·01
7½	37·52	5·36	32·16
8	42·69	6·10	36·59
8½	48·19	6·88	41·31
9	54·02	7·72	46·30
9½	60·20	8·60	51·60
10	66·70	9·53	57·17
10½	71·52	10·22	61·30
11	80·69	11·53	69·16
11½	88·18	12·58	75·59
12	96·00	13·71	82·29
12½	104·17	14·88	89·29
13	112·67	16·09	96·58
13½	121·50	17·36	104·14
14	130·67	18·67	112·00
14½	140·17	20·02	120·15
15	150·00	21·43	128·57
15½	160·17	22·88	137·29
16	170·67	24·38	146·29
16½	181·50	25·93	155·57
17	192·67	27·52	165·15
17½	204·17	29·17	175·00
18	216·00	30·86	185·14
18½	228·17	32·58	195·59
19	240·67	34·38	206·29
19½	253·50	36·21	217·29
20	266·67	38·07	228·60
21	294·00	42·00	252·00
22	322·67	46·09	276·58
23	352·67	50·38	302·29
24	384·00	54·86	329·14
25	416·67	58·09	358·58
26	450·67	64·38	386·29
27	486·00	69·43	416·57
28	522·67	74·67	448·00
29	560·67	80·09	480·58
30	600·00	85·71	514·29

CONTOUR LEVELLING, OR CONTOURING.

WITHIN a comparatively recent period contour levelling, or contouring, has become an important part of the professional knowledge required both by the civil engineer and engineering surveyor. A plan of any city, town, or district, with the contour lines carefully laid down on it, enables the engineer to devise the best and most economic means of carrying out any system of sewage, drainage, irrigation, or waterworks, as well as railway or public road that may be required. By the contour lines the inequalities of the surface of any city, town, or district are correctly represented.

If we imagine a hill to be cut by any number of horizontal planes, and the outline of each cut, as seen from above, to be projected orthographically on the map or plan, the outlines of the cuts so projected are called contour lines. These lines are identical in position with the outlines that would be formed by the sea surrounding the hill, and rising to heights corresponding with those at which the horizontal planes just referred to would cut the hill. If a hill were similar in form to a right cone, its contour lines would be represented on the map or plan by a number of concentric circles; the apex of the cone being the centre, and the outermost circle the circumference of the base of the cone. A hill in shape like an oblique cone would be represented by eccentric circles.

On page 222 is a plan, on a scale of ten chains to the inch, of a part of the Borough of Liverpool, with the contour lines, or lines of equal altitude, represented thereon. The contour lines are shown at every four feet of altitude, as indicated by the numbers inserted on them.

The method generally adopted for determining the position of the contour lines is this: levels are taken along the most suitable streets or roads, or in a direct line between two or more points of the district to be contoured. The most suitable roads, streets, or lines to be levelled, are those which intersect the contour lines at right angles, or nearly so. Bench marks are made in the most favourable places, or stakes are driven in the ground at or near the points where the contour lines will run through; the altitude of each stake being carefully marked on it. Suppose, for instance, that the line from A to B on the plan (p. 222) had been correctly levelled, and that stakes had been driven down at 64 feet above the datum adopted by the Ordnance Survey authorities at 68 feet, at 72 feet, and so on. It is evident the contour lines of corresponding altitudes must pass through the positions of these stakes. Then, in order to determine the 64 feet contour, for example, the level is placed at a distance of 4 or 5 chains from the stake at that altitude, the back staff is placed on the stake, and read off by the level, after it has been carefully levelled;

the staff is then sent forward 4 or 5 chains in the direction the contour line seems to take, and placed on the ground; if the reading of it by the level agrees with what it had been on the stake, the position of the staff is in the contour line. If the reading of the staff should be more or less than that at the stake, the staff is to be moved to a higher or lower position, until the reading is the same as that at the stake. When the true position is found, the level is then moved forward, while the staff remains where it is until the back-sight or reading is taken, when the staff is again sent forward, and so on. Previously, however, to the staff being sent forward, some distinctive mark should always be placed where it stood, in order that the surveyor may be enabled to survey the contour lines, so as to fix their positions on the map or plan.

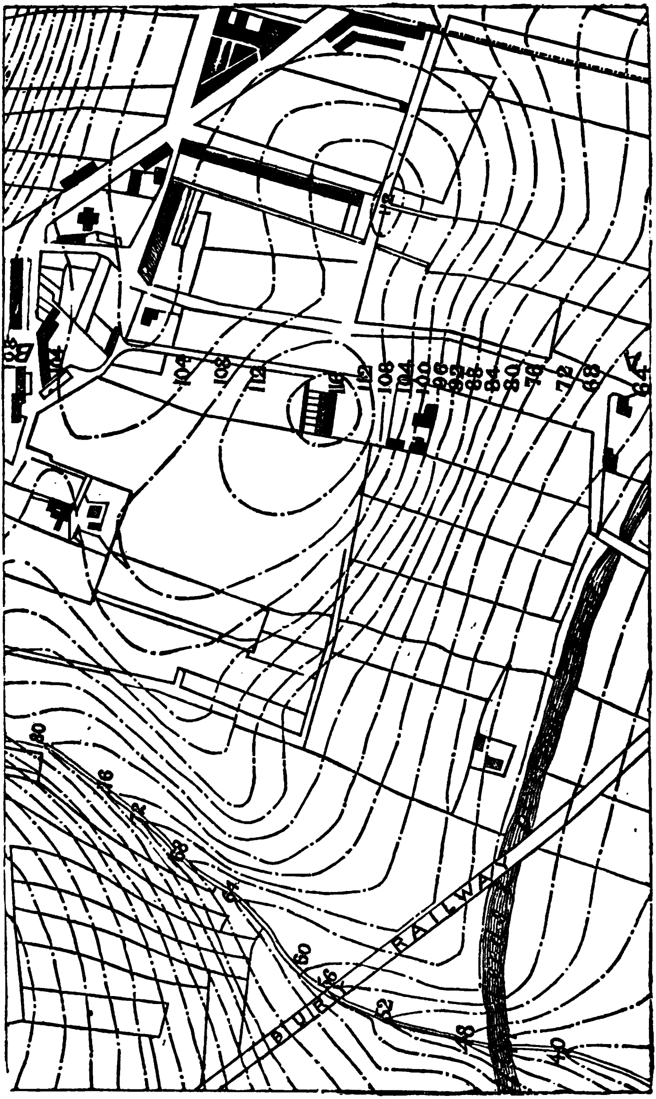
The marks usually placed at the positions of the staff on the contour lines, are twigs, or small cuttings off the branches of trees. These are stuck in the ground, with pieces of paper at the top, to render them more conspicuous. The surveyor then lays out his chain lines in the most suitable manner, and takes offsets to the twigs or cuttings, and notes them in his field-book. He is then able to plot the various courses of the contour lines as if they were fences, and to show, if necessary, the position on the map or plan of every point on the contour line where the staff stood. The dots on the contour lines, on the plan, p. 222, represent the positions of the staff. If the map or plan be correct, the chain lines for surveying the contours can generally be fixed by means of the fences and other details.

The writer of these remarks prepared contour plans of the city of Londonderry, and the town of Preston, Lancashire, for their respective Corporations, preliminary to the carrying out of the sewerage and drainage of those places. The contour lines were shown at altitudes differing only five feet from each other.

On the Ordnance Survey of Great Britain, the lowest contour is run at 50 feet above the Ordnance datum, which datum is the height of mean tide at Liverpool, as determined by careful tidal observations taken in 1844. The next contour is run at 100 feet, then at 200, 300, and at intervals of 100 feet up to an altitude of 1,000 feet. Above this, no instrumental contours have been run, but sketched contours are shown at intervals of 250 feet.

The datum plane adopted for the whole of the Ordnance Survey of Ireland was the level of low water spring tide observed at Poolbeg Light-house in Dublin Bay on the 8th April, 1837, which level was found by a combined system of tidal observation and spirit levelling to be 8·094 feet below the mean sea level.

Seven counties in Ireland have been instrumentally contoured by the Ordnance Survey, but the vertical interval between the contours varies in different localities.



104
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48
40
BLUE
RAILWA

ADDENDA TO THE METHODS OF LAYING OUT RAILWAY CURVES.

Problem I.

To join the straight portions A B, C D *of a railway by a serpentine curve* F O I N M, *meeting* A B *at* F *between* A *and* B, *and* C D *at* M *between* C *and* D; *the distance* B C *and the* ∠s A B C, B C D *being given, to find the common radius* G I = I H.

Take the cotangents of half the angles A B C, B C D to rad. = 1; then as the sum of these two cotangents is to the cotangent of half A B C to rad. = 1, so is B C to B I; hence B C − B I = I C.

In the triangle B C I; as sin. ∠ B G I is to sin. ∠ G B I, so is B I to G I = I H, the common radius.

Example.

Let the angle A B C = 71° 40′, the angle B C D = 129° 15′, and the distance B C = 95 chains: required the length of the radius G I = I H of the serpentine curve for uniting A B to C D.

Here the cotangents of half the angles A B C, B C D to rad. = 1 are respectively 1·3848 and ·4743, their sum being 1·8591. Then as 1·8591 : 1·3848 :: 95 : 70·76 chains = B I. Whence B C − B I = 95 − 70·76 = 24·24 chains = I C.

Again, as sin. B G I is to sin. G B I, so is 70·63 to 51·09 chains = G I = I H, which is the common radius of the required serpentine curve.

Problem II.

To lay out a railway curve by means of tangental angles.

This method, already noticed in the note to Problem IV., p. 169, is preferred by many eminent engineers, where the ground on which the curve is to be laid out is level and free from numerous obstructions, except such as do not fall on the curve itself; I shall therefore give it more in detail than in the note referred to.

If from any point B, in a straight line A D, we lay off any number of equal angles, as D B S, S B T, T B U, &c., making the chords B S, S T, T U, &c., equal to each other; then the points B, S, T, U, &c., will be situated in the circum-

ference of a circle, which is tangental to the line A D at the point B.

Fix the theodolite at B, and lay off the tangental angle D B S, the chain being extended from B to S to meet the visual line B S; in the same manner lay off the other angles S B T, T B U, &c., till the fourth point V is reached. If any obstruction, as H, should prevent our seeing from B further than to V, the curve may be continued by removing the instrument to U, the point preceding V; thence sighting first on V, continue to lay off additional tangental angles V U W, W U X, &c., as before. Otherwise, moving the instrument to V instead of U, sight back to U, and lay off first the exterior angle P V W equal to *double* the tangental angle (P being in U V prolonged), and afterwards continue the tangental angles W V X, X V Y, &c., as before, to the end of the curve.

Finally, in order to pass to the end of the curve at Y to a tangent Y Z, place the instrument at Y, and sighting back to X. lay off the tangental angle X Y O; then O Y continued towards Z will be the required tangent to the curve.

The method of finding the quantity of the tangentil angles is given in the note, p. 166

ON THE DIVISION OF LAND.

Three straight fences W X, X Z, Z Y are given in position; there is a well at W, and it is required to substitute a new straight fence W Y instead of X Z, that the ground on both sides of the divisional fence may have the use of the well; the value of the land on the side W X Z is to that on the side X Z Y as *m* to *n* per acre: required the position of the new fence W Y, that the proprietors on each side of it may neither gain nor lose by the change of boundary.

Let the required straight fence W Y cut X Z in P; draw W Q parallel to Y Z, meeting X Z in Q; then Q is a given point, and the area W Q X is also given, which put $=$ A, also put Z Q $= a$, Q W $= b$, Q P $= x$, and the sine of the angle Z $=$ sine of the angle Z Q W $= \theta$; then from the nature of the problem we readily obtain the following quadratic equation:

$$x^2 + \frac{2(n a b \theta + \mathrm{A} m)}{b \theta (m - n)} x - \frac{n a^2 b \theta}{b \theta (m - n)} = 0.$$

From this equation the value of $x = \mathrm{QP}$ may be readily found, which determines the position of the point P, through which the new fence W Y must pass.

The quantity A must be regarded as positive or negative, accordingly as the point Q falls in the fence X Z, or in its prolongation to the other side of X W.

Corollary 1.—When the land on both sides of X Z is of equal value; then $m = n$, and the second power of x in the preceding equation will vanish, and

$$\mathrm{QP} = x = \frac{a^2 b \theta}{2(a b \theta \pm \mathrm{A})}.$$

Corollary 2.—When Y Z is parallel to X W, Q will coincide with X, and A will vanish; whence

$$\mathrm{QP} = x = \tfrac{1}{2} a = \tfrac{1}{2} \mathrm{XZ}.$$

NOTE. Changes of boundaries of this kind are frequently required, not only for the advantages of wells or watering places, but also for ready access to more convenient roads, &c. This important article was inadvertently omitted in the previous editions of this work.

INDEX.

THE END

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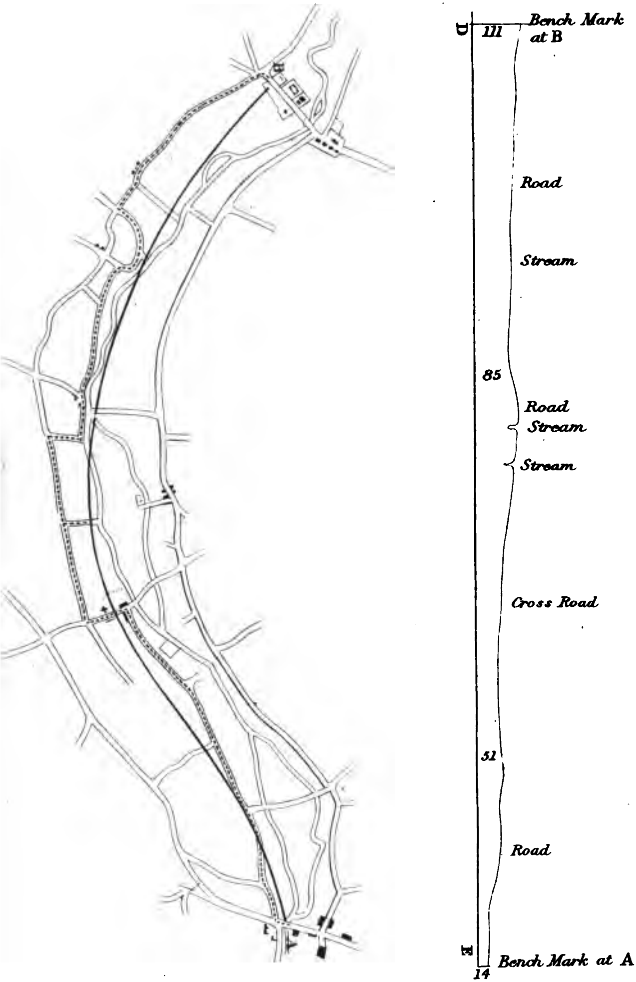
PLATE 1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at B
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Stream
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Stream
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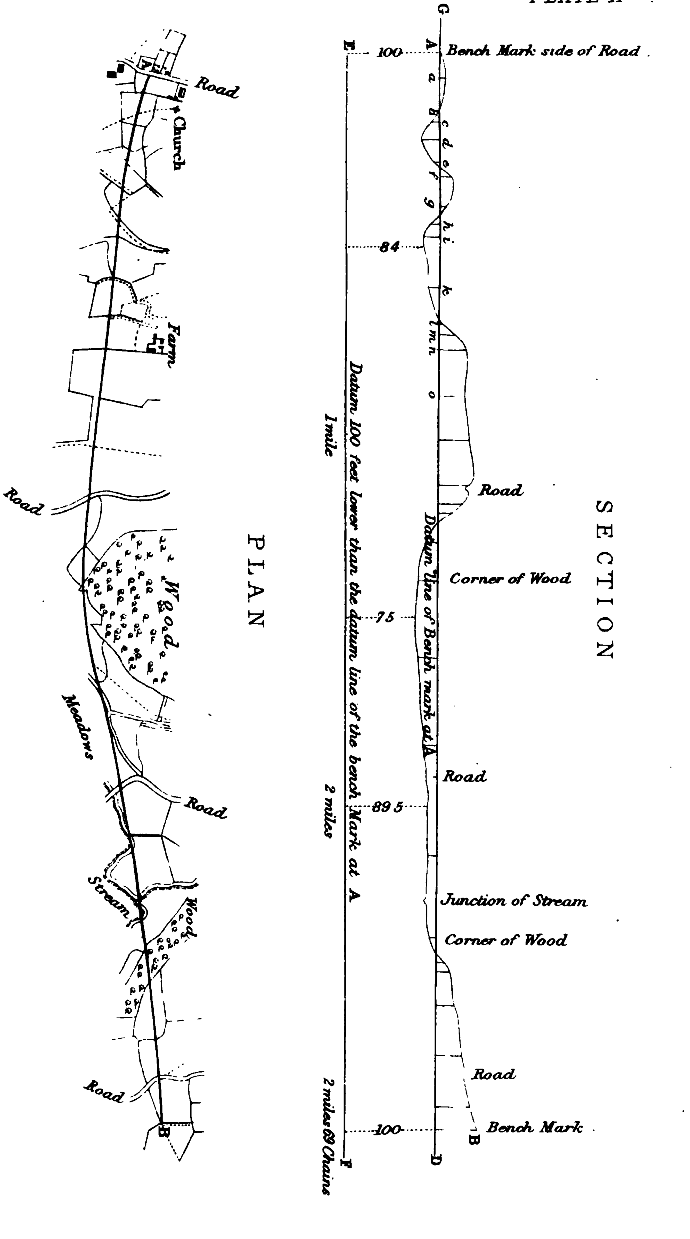
Road
E
Bench Mark at A
14


PLAN
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Road
Church
Farm
Road
Wood
Meadows
Road
Stream
Wood
Road
A
B
G
A
E
100
Bench Mark side of Road
a
b
c
d
e
f
g
h
i
k
l
m
n
o
84
Road
Datum line of Bench mark at A
Corner of Wood
75
Road
89 5
Junction of Stream
Corner of Wood
Road
100
Bench Mark
B
D
F
Datum 100 feet lower than the datum line of the bench Mark at A
1 mile
2 miles
2 miles 69 Chains